Exclusion limits on heavy neutral MSSM Higgs bosons A/H decaying to a pair of top quarks in pp collisions at $\sqrt{s} = 13$ TeV in the LHC

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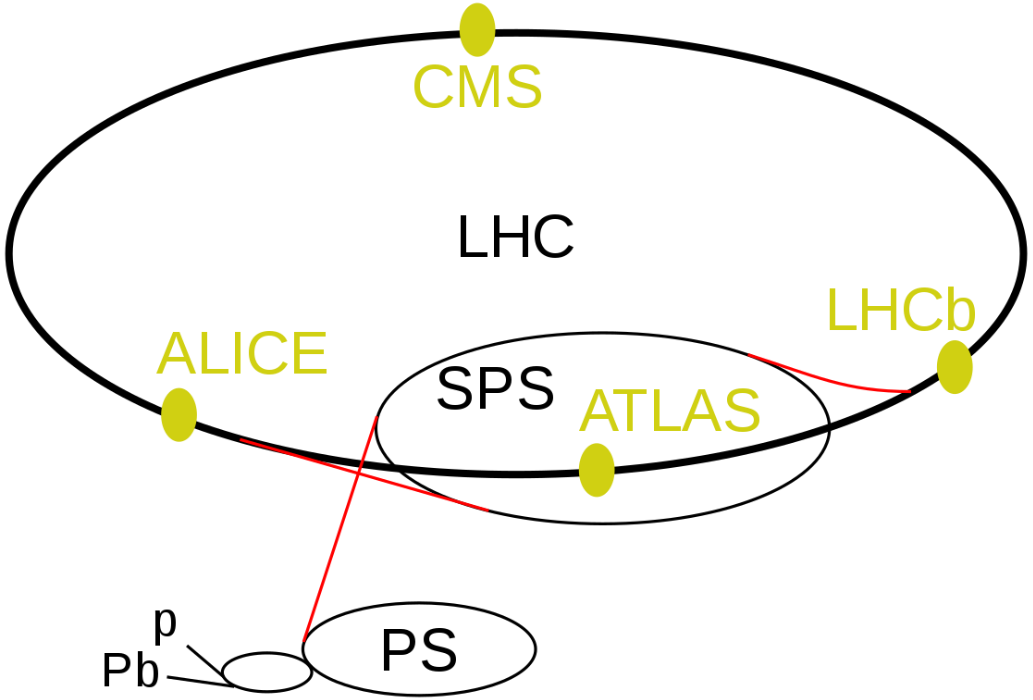
The Large Hadron Collider (LHC) experiment

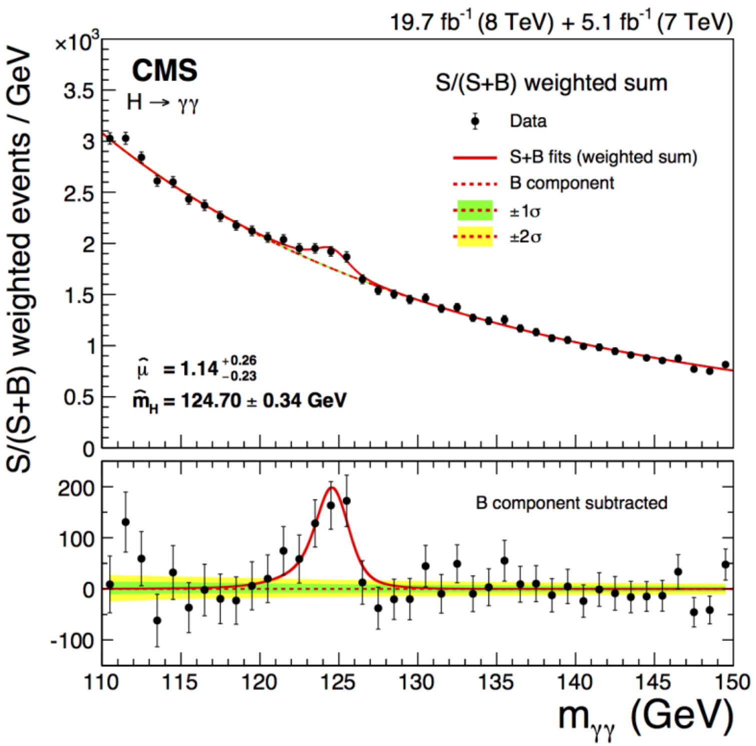


Main Objectives

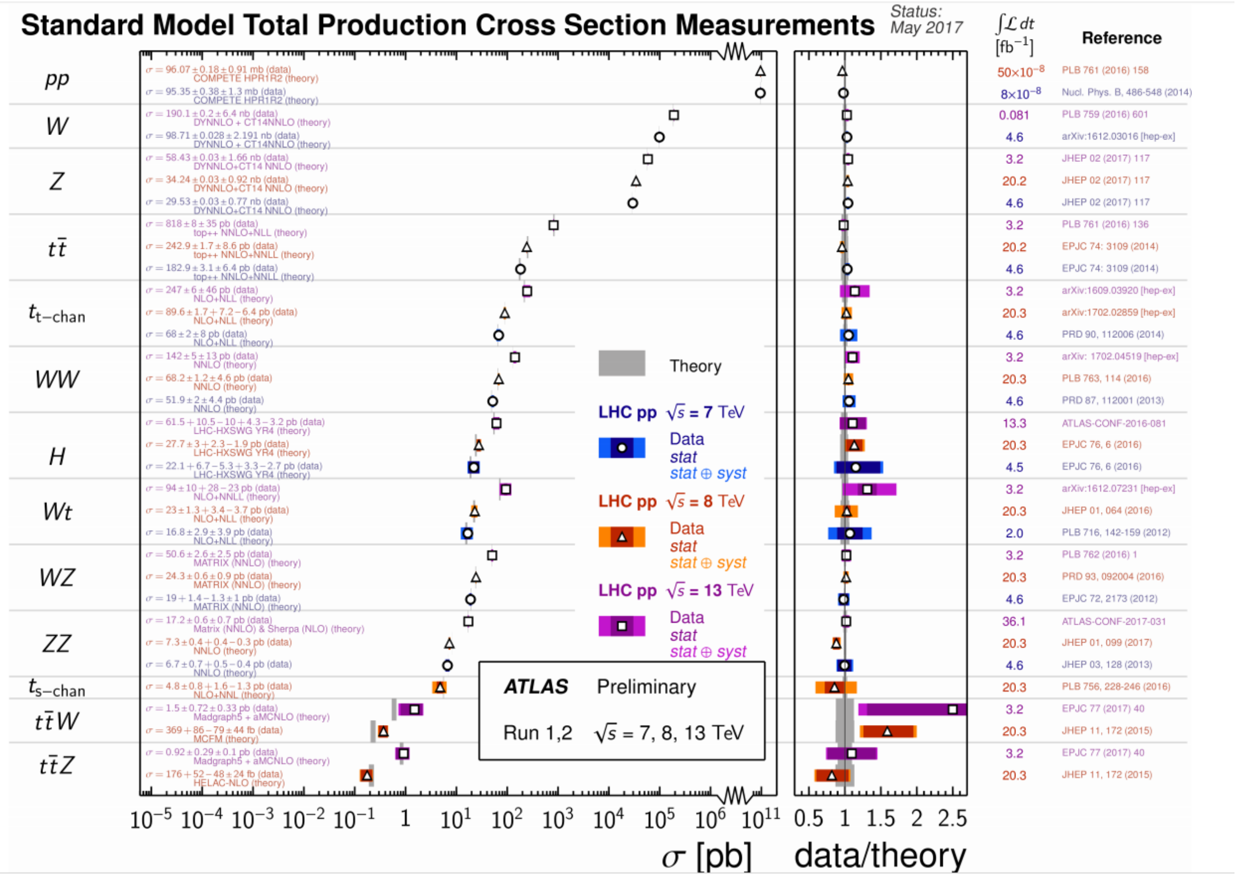
- Search for the Higgs boson
- Investigate different scenarios of physics beyond the Standard Model (BSM)
- Perform precision measurements of Standard Model processes

The Triumph of the Standard Model (SM)

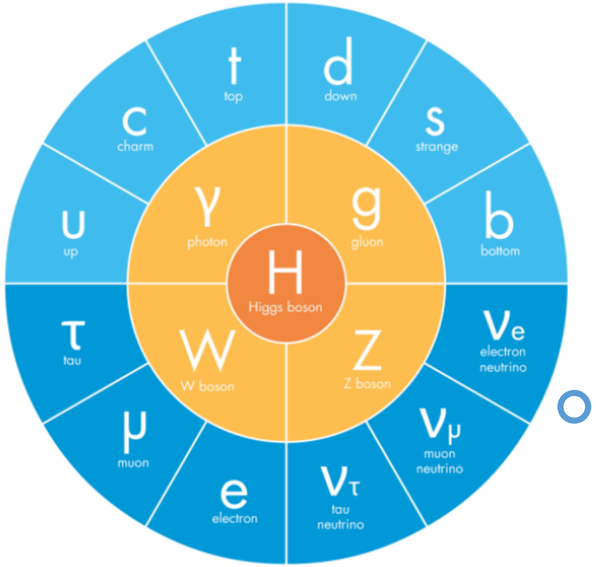
- Found the Higgs boson!



- Precision measurements consistent with SM.

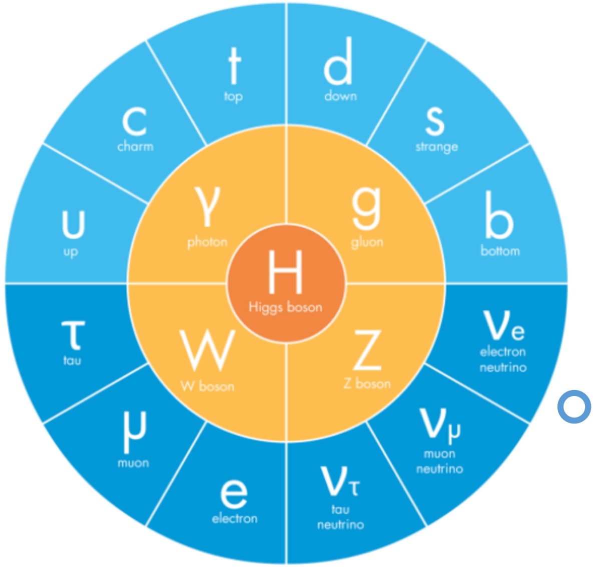


The Triumph of the Standard Model (SM)



Is this the end of the story?

The Triumph of the Standard Model (SM)

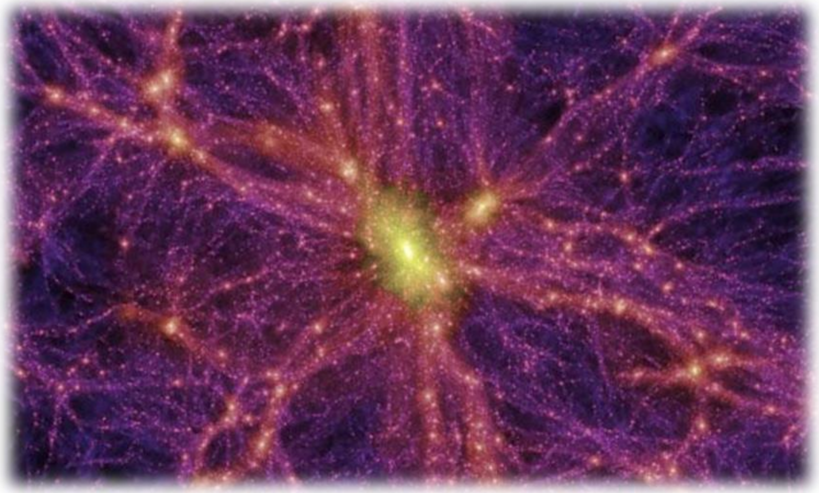


Is this the end of the story?

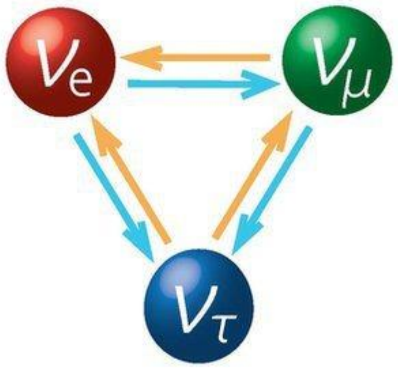
NO! The Standard Model is INCOMPLETE

The Triumph of the Standard Model (SM)

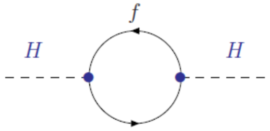
- It does not explain Dark Matter (DM)

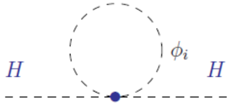


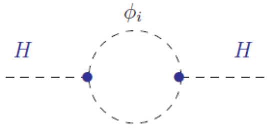
- It does not explain the mass of neutrinos.



- The naturalness (fine-tuning) and hierarchy problem:
  Quadratically divergent behavior in the radiative corrections to the SM Higgs boson mass



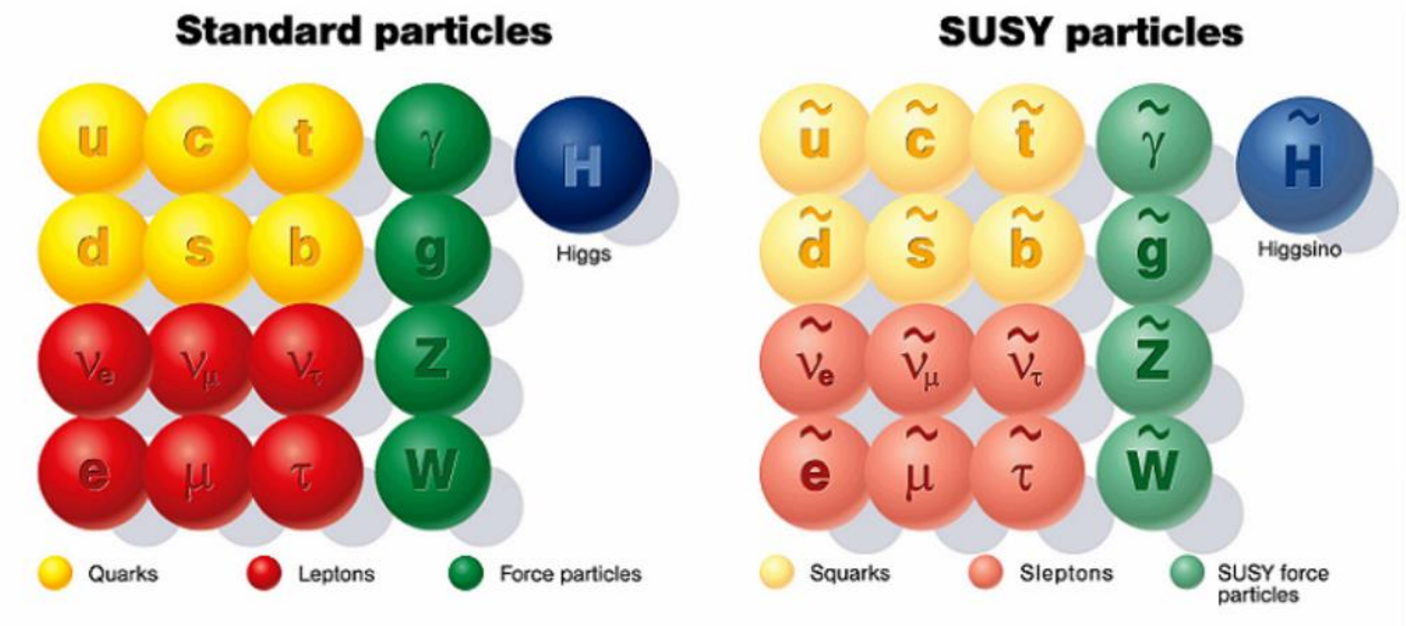




$$\Delta M_H^2 = N_f \frac{\lambda_f^2}{8\pi^2}\left[-\Lambda^2 + 6m_f^2\log\frac{\Lambda}{m_f} - 2m_f^2\right] + \mathcal{O}(1/\Lambda^2)$$

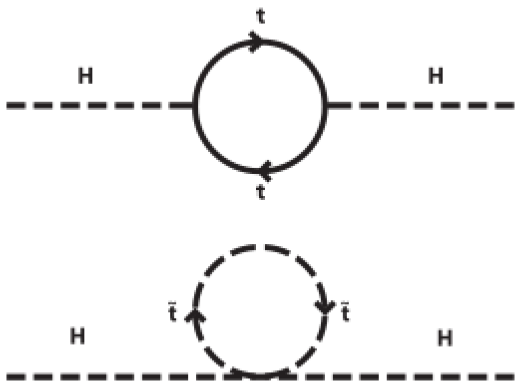
Hierarchy problem: Why $\Lambda \gg M_Z$

Supersymmetry (SUSY) Models



- Invokes a symmetry between bosons and fermions with the introduction of heavier superpartners to each elementary particle

Supersymmetry (SUSY) Models



- It allows for the cancellation of radiative corrections
- In the minimal SUSY scenario (main focus of this research)

One can introduce a discrete symmetry: R-parity

which enforces lepton and baryon number conservation

→Lightest SUSY particle is absolutely stable

1. This is the lightest of the four neutralinos, which is massive, electrically neutral and weakly interacting.
2. It can have the right cosmological relic density to account for the cold Dark Matter in the universe

Minimal Supersymmetric Standard Model (MSSM)

- Minimal gauge group and particle content

$$\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$$

Same as SM

- Two Higgs doublet field, Φ_1 and Φ_2, are required to break the electroweak symmetry

- Five Higgs bosons in the MSSM Higgs sector:

  - 2 charged : $H^{\pm}$

  - 2 neutral scalars : h, H

  - 1 neutral pseudoscalar : A

Heavy neutral Higgs bosons

- Only two input parameters are needed:

  1. Mass of pseudoscalar: M_A

  2. Ratio of vacuum expectation values of the Higgs doublet field: $\tan\beta$

The $gg \to A/\mathrm{H} \to t\bar{t}$ decay channel

- Two efficient decay channels for heavy netural MSSM Higgs bosons search:

  1.$\mathrm{gg} \to A/\mathrm{H} \to \tau^+\tau^-$: Suitable for probing high $\tan\beta$ regime

  Reason: The existence of a second Higgs doublet field →

  Strong coupling enhancement to bottom quark associated production and the decay to taus

  at high $\tan\beta$

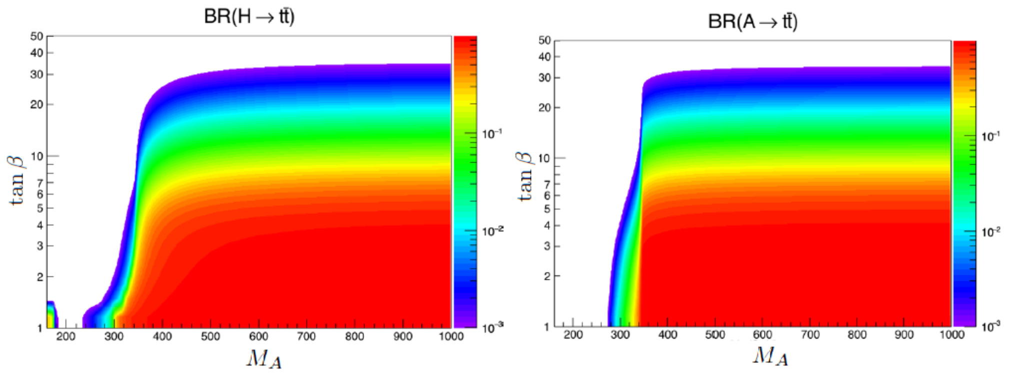
  2. $gg \to A/\mathrm{H} \to t\bar{t}$: Suitable for probing low $\tan\beta$ regime

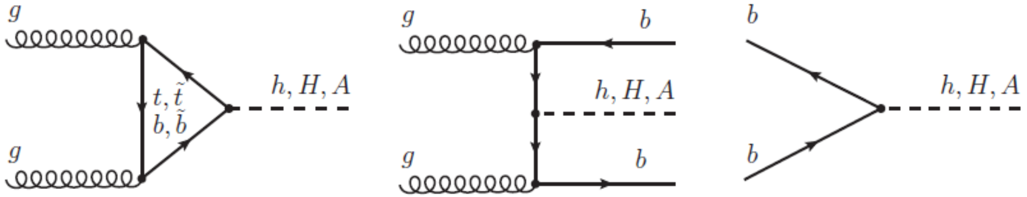
  Reason: Strong top-quark (most massive elementary particle) Yukawa coupling $\propto m_t\ /\tan\beta$

- Particular focus on the pseudoscalar A because

  $A \to WW/ZZ$ decays are forbidden due to CP conservation

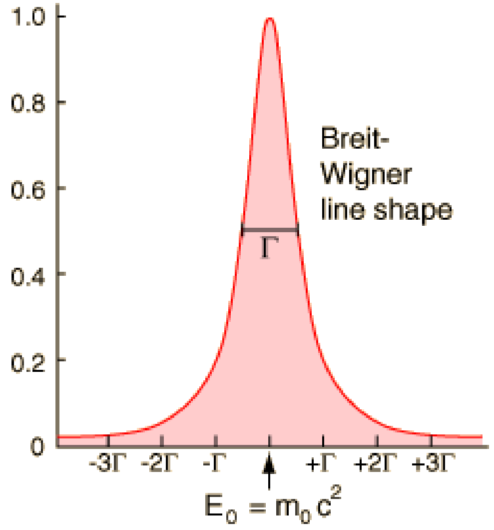
  $\mathrm{A} \to t\bar{t}$ will be the only channel to find A at low $\tan\beta$





Prelude

- Coupling, g, ~ Strength of an interaction
- Cross-section, σ, ~ Probability for an event to happen
- Luminosity, L, ~ Number of events per second for a given cross section
- Branching ratio (for a decay) ~ Fraction of particles which decay by a particular decay mode
- Decay width, Γ, = 1 / mean life time ~ width of mass resonance



hMSSM Higgs sector

- MSSM coupling can be expressed in terms of $\tan\beta$ and the mixing angle α

Φ	$g_{\Phi\bar{u}u}$	$g_{\Phi\bar{d}d}$	$g_{\Phi\bar{V}V}$
H_{SM}	1	1	1
h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
A	$\cot\beta$	$\tan\beta$	0

- The coupling plays a central role in determining the cross-sections and decay widths for a particular process. It is used as a parameter for setting upper limits in MSSM

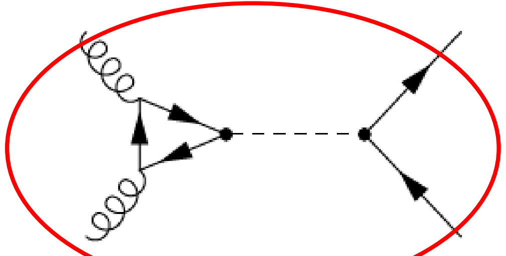
- The scalar Higgs mass and the mixing angle is related to $\tan\beta$ and M_A by

$$M_H^2 = \frac{(M_A^2 + M_Z^2 - M_h^2)(M_Z^2\cos^2\beta + M_A^2\sin^2\beta)}{M_Z^2\cos^2\beta + M_A^2\sin^2\beta - M_h^2} - \frac{M_A^2 M_Z^2\cos^2 2\beta}{M_Z^2\cos^2\beta + M_A^2\sin^2\beta - M_h^2}$$

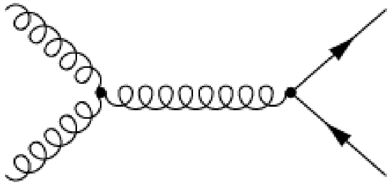
$$\alpha = -\arctan\left(\frac{(M_Z^2 + M_A^2)\cos\beta\sin\beta}{M_Z^2\cos^2\beta + M_A^2\sin^2\beta - M_h^2}\right)$$

Signal and Interference

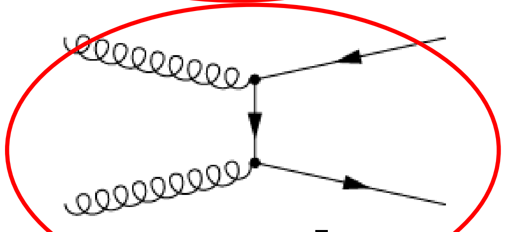
- Interference phenomenon occurs because the initial and final states of the Higgs production process are identical to the $t\bar{t}$ production
- Interference occurs between the gluon-gluon initiated loop production and the SM $t\bar{t}$ production
- It causes the signal shape to distort from a simple Breit-Wigner peak to a peak-dip structure
- Interference effect puts limitation on the sensitivity of the search for the signal resonance



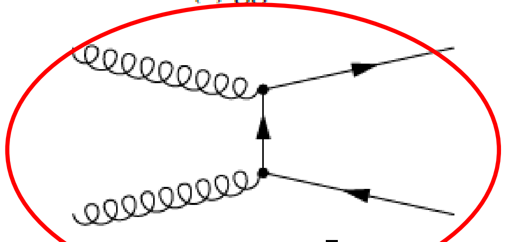
(a) $gg \to \Phi \to t\bar{t}$



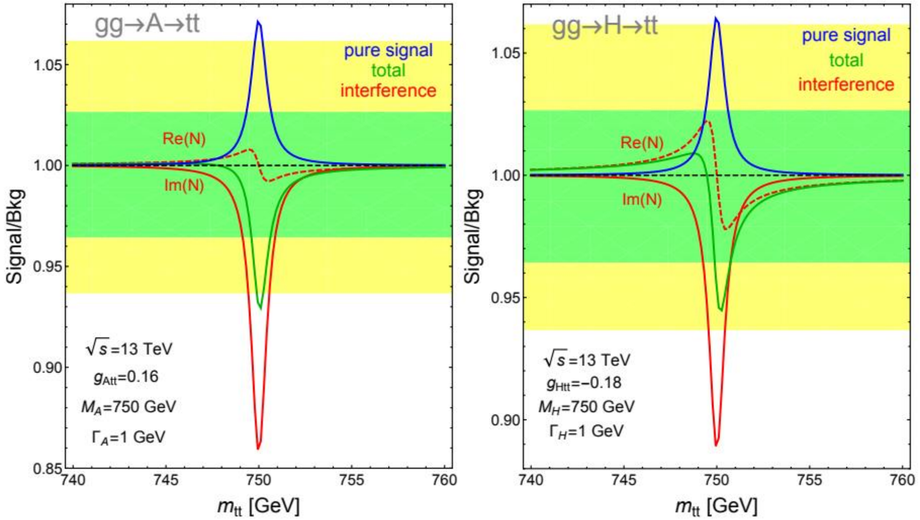
(b) $gg \to t\bar{t}$



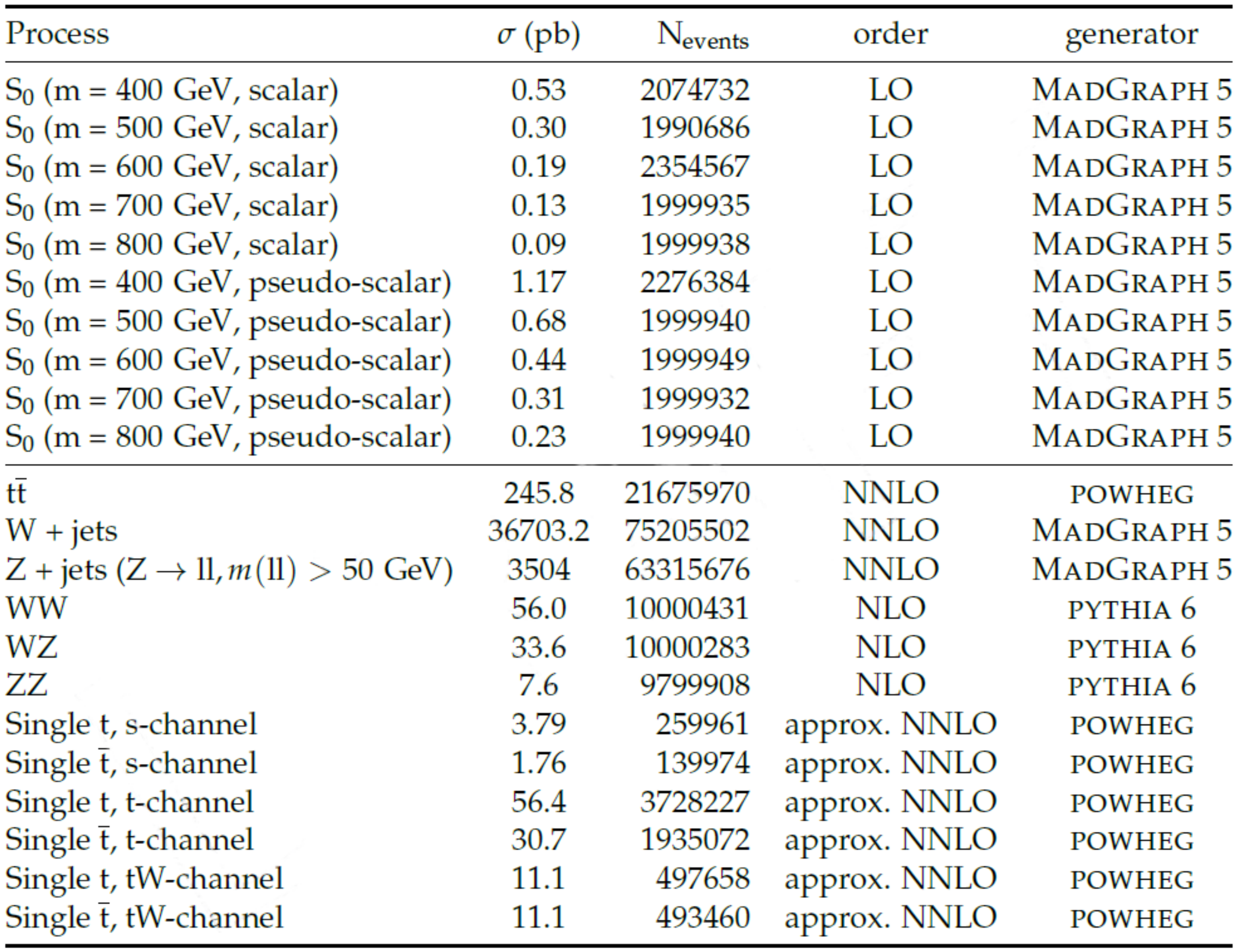
(c) $gg \to t\bar{t}$



(d) $gg \to t\bar{t}$



Event Generation and K-factor



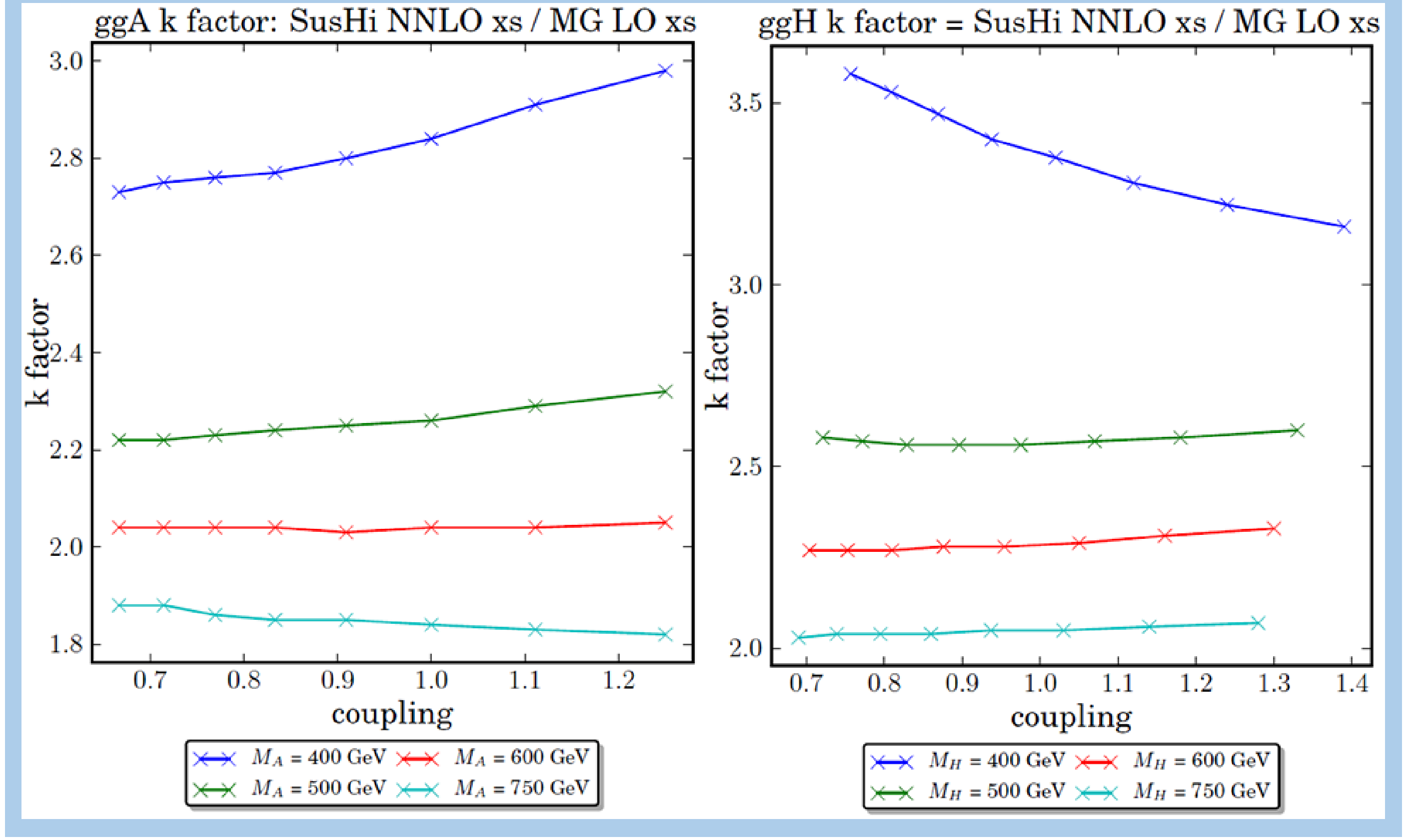
Process	σ (pb)	N_{events}	order	generator
S_0 (m = 400 GeV, scalar)	0.53	2074732	LO	MADGRAPH 5
S_0 (m = 500 GeV, scalar)	0.30	1990686	LO	MADGRAPH 5
S_0 (m = 600 GeV, scalar)	0.19	2354567	LO	MADGRAPH 5
S_0 (m = 700 GeV, scalar)	0.13	1999935	LO	MADGRAPH 5
S_0 (m = 800 GeV, scalar)	0.09	1999938	LO	MADGRAPH 5
S_0 (m = 400 GeV, pseudo-scalar)	1.17	2276384	LO	MADGRAPH 5
S_0 (m = 500 GeV, pseudo-scalar)	0.68	1999940	LO	MADGRAPH 5
S_0 (m = 600 GeV, pseudo-scalar)	0.44	1999949	LO	MADGRAPH 5
S_0 (m = 700 GeV, pseudo-scalar)	0.31	1999932	LO	MADGRAPH 5
S_0 (m = 800 GeV, pseudo-scalar)	0.23	1999940	LO	MADGRAPH 5
$t\bar{t}$	245.8	21675970	NNLO	POWHEG
W + jets	36703.2	75205502	NNLO	MADGRAPH 5
Z + jets ($Z \rightarrow ll, m(ll) > 50$ GeV)	3504	63315676	NNLO	MADGRAPH 5
WW	56.0	10000431	NLO	PYTHIA 6
WZ	33.6	10000283	NLO	PYTHIA 6
ZZ	7.6	9799908	NLO	PYTHIA 6
Single t, s-channel	3.79	259961	approx. NNLO	POWHEG
Single $\bar{t}$, s-channel	1.76	139974	approx. NNLO	POWHEG
Single t, t-channel	56.4	3728227	approx. NNLO	POWHEG
Single $\bar{t}$, t-channel	30.7	1935072	approx. NNLO	POWHEG
Single t, tW-channel	11.1	497658	approx. NNLO	POWHEG
Single $\bar{t}$, tW-channel	11.1	493460	approx. NNLO	POWHEG

- To evaluate the expected limits for MSSM, MC simulated data samples generated with respect to the background only hypothesis are fitted against the observed data (or the Asimov data set)
- All events are generated at the center-of-mas energy of 13 TeV in pp collisions
- Aspect of event reconstruction and selection will not be discussed
- Signal and interference events are generated using MadGraph5 aMC@NLO at LO only to save computing time (cross section for background events are 2 orders larger than signal events)
- To include higher order QCD corrections to the signal and interference cross-section, a rescaling of signal and interference events by a K-factor is applied

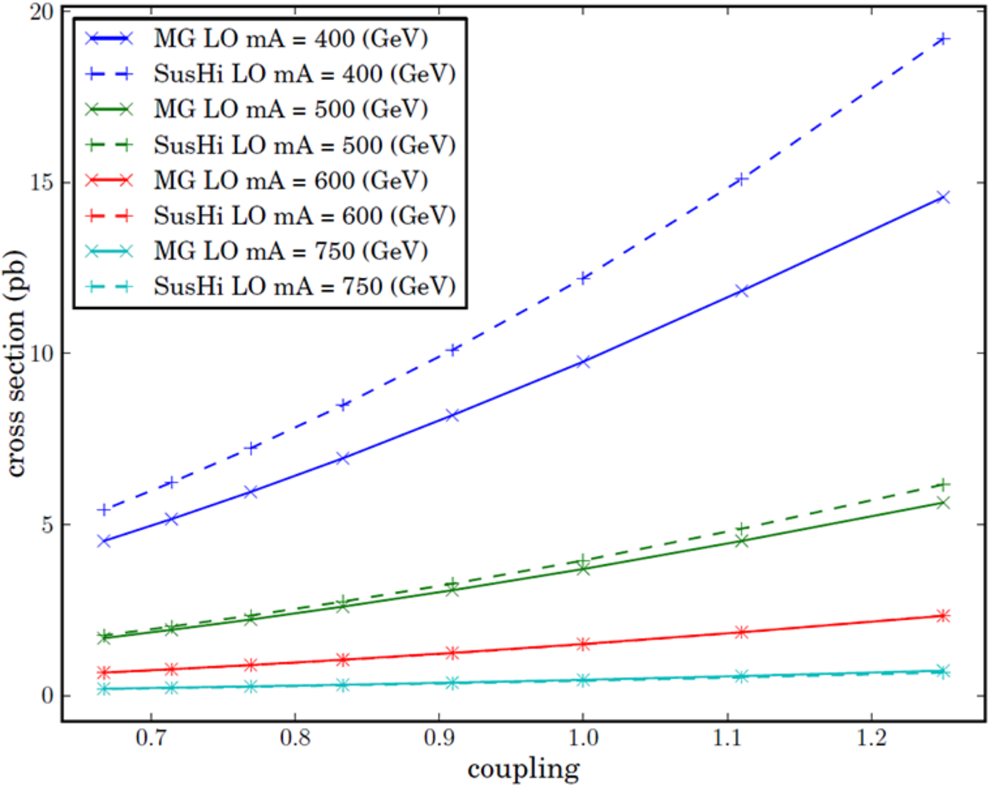
Event Generation and K-factor

$$\text{K-factor} = \frac{\text{SusHi NNLO xs}}{\text{MG LO xs}}$$

- For simplicity, the value at coupling = 1 is used

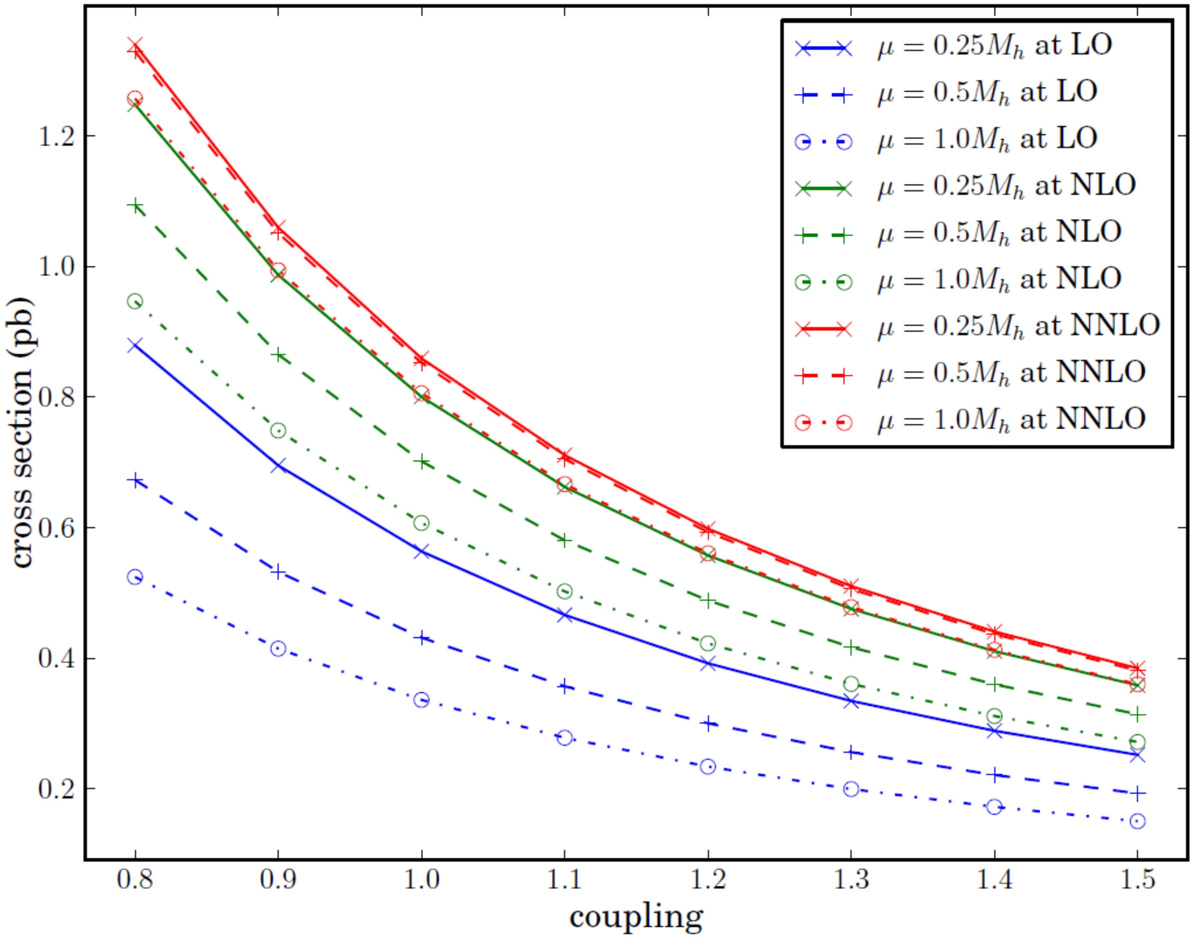


- Some inconsistency observed at LO xs:



Appendix: Renormalization and Factorization scales

- The K-factor is highly dependent on the choice of the renormalization and factorization scales, μ_R and μ_F, in the perturbative QCD calculation
- The difference in the calculated cross-sections at different scales will decrease as higher order corrections are included (i.e. at NLO and NNLO)



Mass and Width Morphing

- We generate data samples in with Higgs masses between [400,750] GeV (most sensitive region) and with widths between [2.5,50] percent, with respect to the Higgs masses M_A or M_H
- To refine the binning of the simulated data (Save computing time) to 50 GeV mass and 0.5 percent width, mass and width morphing algorithms are implemented
- Mass morphing algorithm: NonLinearPosFractions implemented in RooMomentMorph of ROOT (Too complicated, not disussed)
- Width morphing algorithm:

  Signal: Hyperbolic interpolation   Interference: Linear interpolation

$$\sigma_S^{\text{hMSSM}} \propto \sigma_S^{\text{data}} \cdot g^4 \qquad \frac{m_t^2}{} \hat{s}^2 \sum_{\Phi} \qquad \sigma_S^{\text{data}} \propto \frac{\sigma_S^{\text{hMSSM}}}{g^4} \propto \frac{1}{g^2} \propto \frac{1}{\Gamma} \qquad \frac{2(\hat{\tau}_Q)|^2}{\Gamma_\Phi^2}$$

$$\Gamma(\Phi \to t\bar{t}) = N_c \frac{G_F m_f^2}{4\sqrt{2}\pi} \hat{g}^2_{\Phi tt} M_\Phi \beta_t^{p\Phi}$$

Mass and Width Morphing

- The cross-section for signal goes as coupling^2

$$\frac{d\hat{\sigma}_S}{dz} = \frac{3\alpha_s^2 G_F^2 m_t^2}{8192\pi^3}\hat{s}^2 \sum_{\Phi} \frac{|\hat{\beta}_t^{p\Phi}\hat{g}_{\Phi tt}\sum_Q \hat{g}_{\Phi QQ} A^{\Phi}_{1/2}(\hat{\tau}_Q)|^2}{(s - M_\Phi^2)^2 + \Gamma_\Phi^2 M_\Phi^2}$$

- The decay width also goes as coupling^2

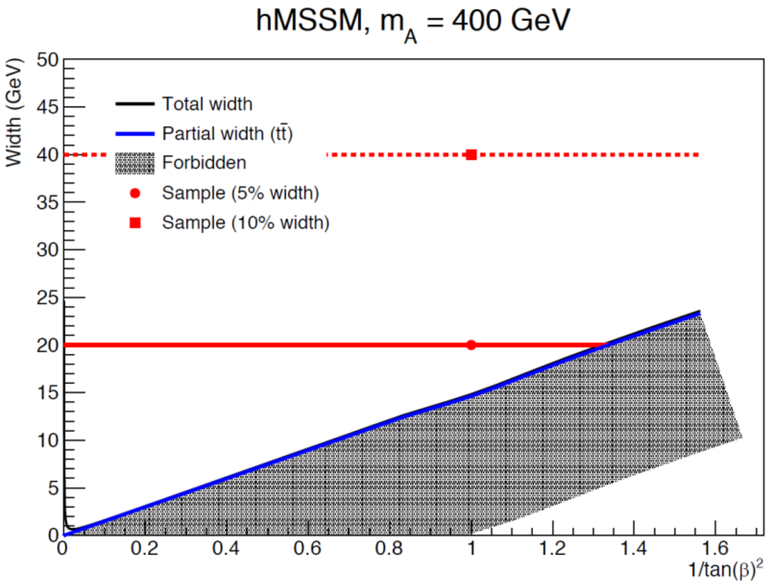
$$\Gamma(\Phi \to t\bar{t}) = N_c \frac{G_F m_f^2}{4\sqrt{2}\pi}\hat{g}^2_{\Phi tt} M_\Phi \beta_t^{p\Phi}$$

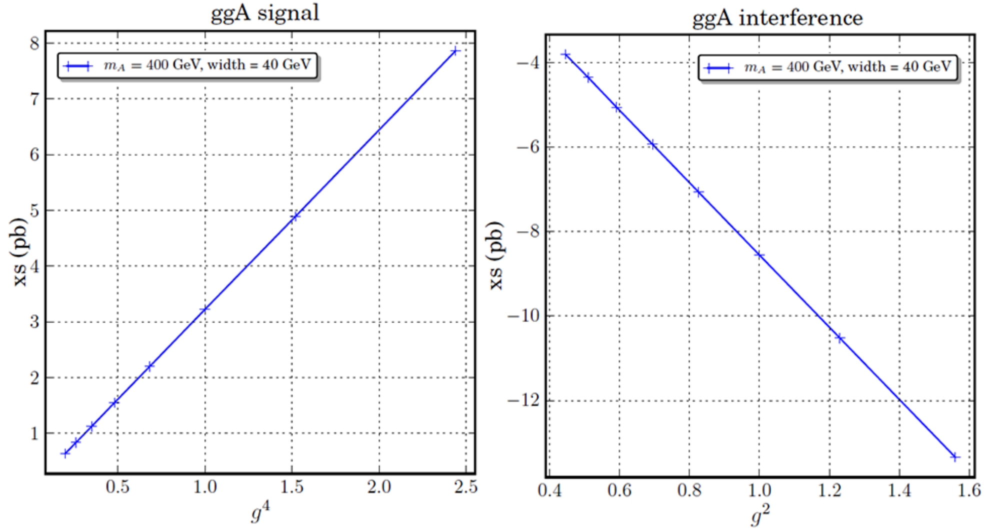
- Along a fixed width, the cross-section for the data (generated according to the background only hypothesis, i.e. g = 1) will go as

$$\sigma_S^{\mathrm{hMSSM}} \propto \sigma_S^{\mathrm{data}} \cdot g^4$$

- The interference amplitude is usually the square root of the amplitudes of the processes that interfere:

$$\sigma_I^{\mathrm{hMSSM}} \propto \sigma_I^{\mathrm{data}} \cdot g^2$$



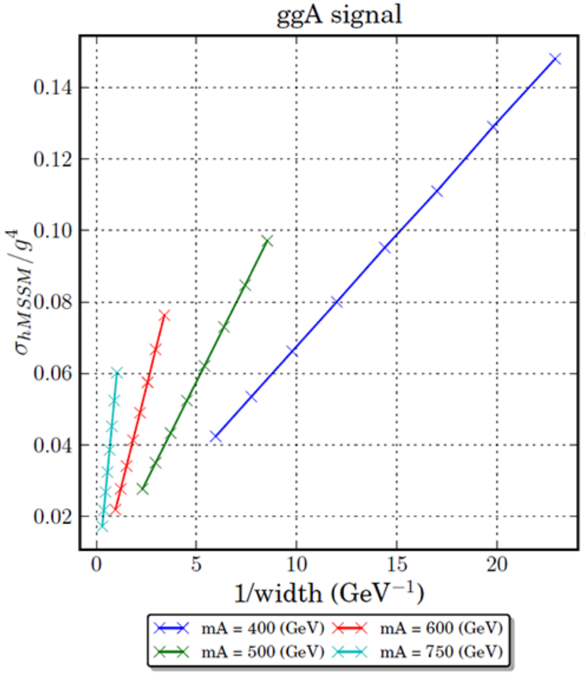


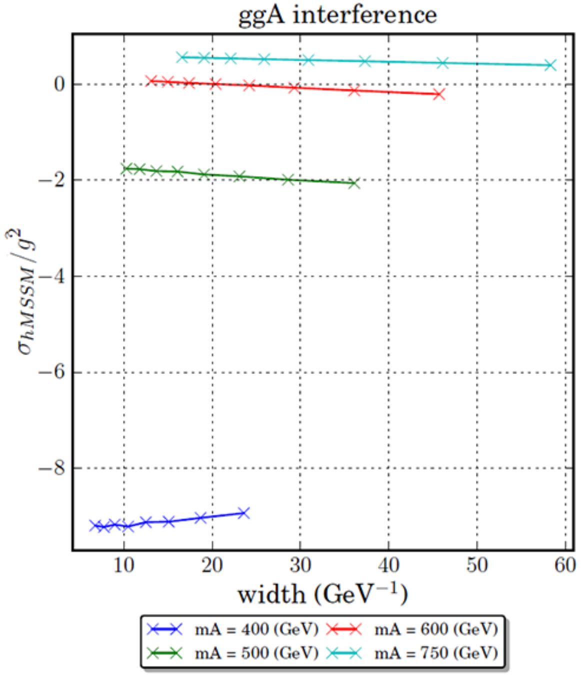
Mass and Width Morphing

- Therefore the cross-section for the signal data will go as

$$\sigma_S^{\mathrm{data}} \propto \frac{\sigma_S^{\mathrm{hMSSM}}}{g^4} \propto \frac{1}{g^2} \propto \frac{1}{\Gamma}$$

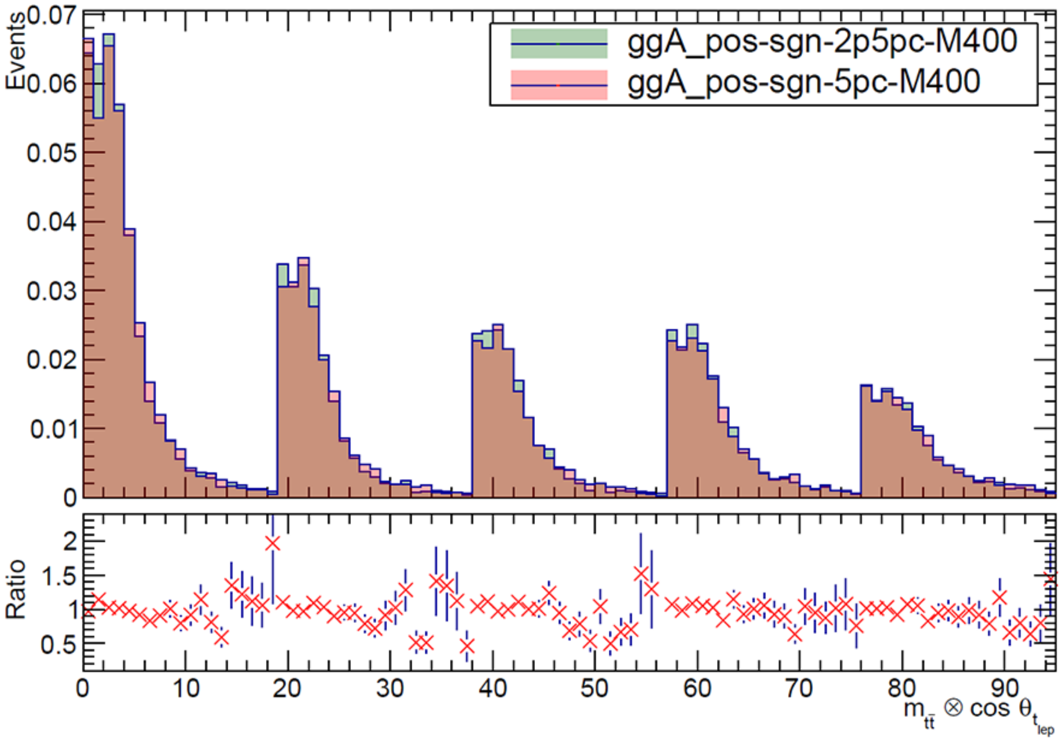
- The relation implies a hyperbolic interpolation scheme should be used.
- For interference events, no simple expressions are found and a linear interpolation scheme is used by default

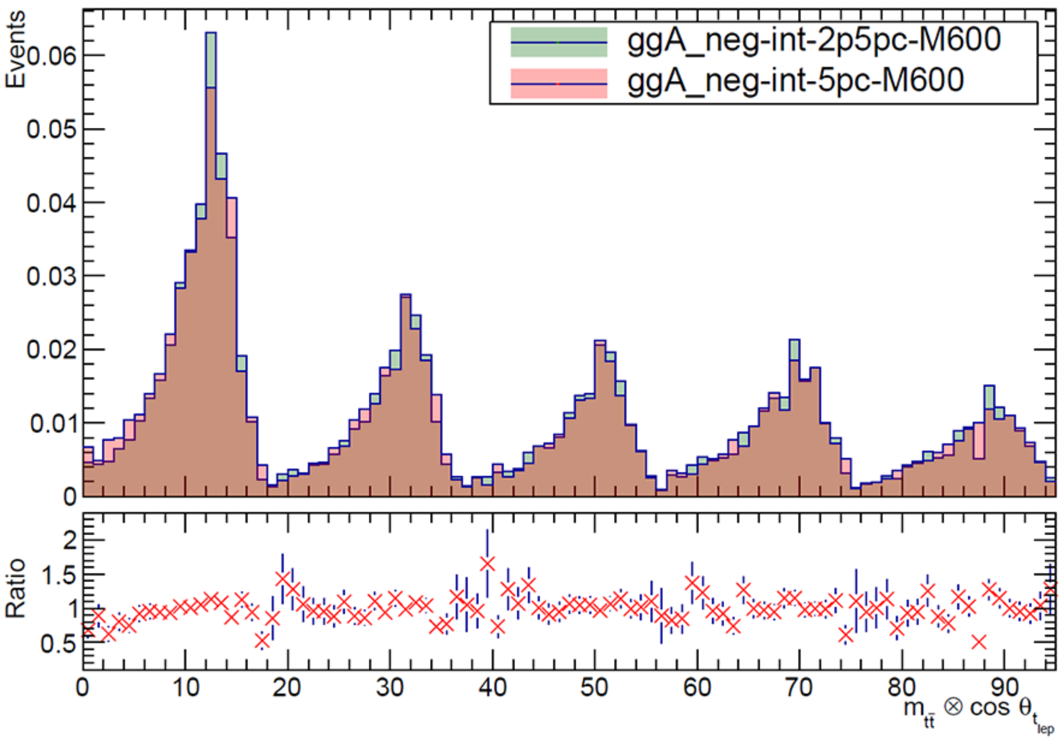




Extrapolation to 1pc width from shapes at 2.5pc width

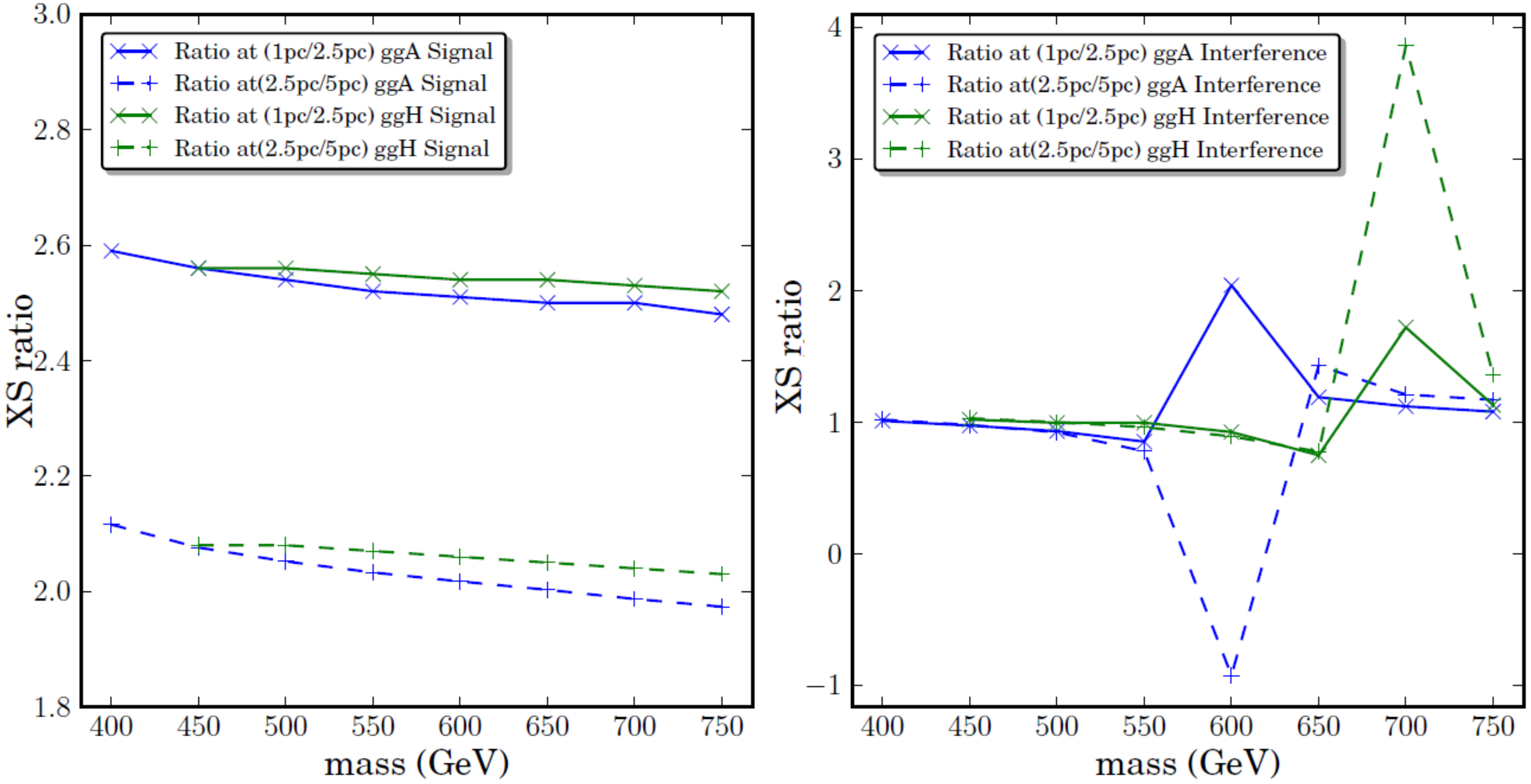
- For evaluation of expected limits, data samples with small widths ~ 1pc are required
- An extrapolation scheme is proposed which scales the signal shapes at 2.5pc according to the ratio of cross-sections obtained at 2.5 pc and 1pc width
- Comparison of signal and interference shape at 2.5pc and 5pc width:





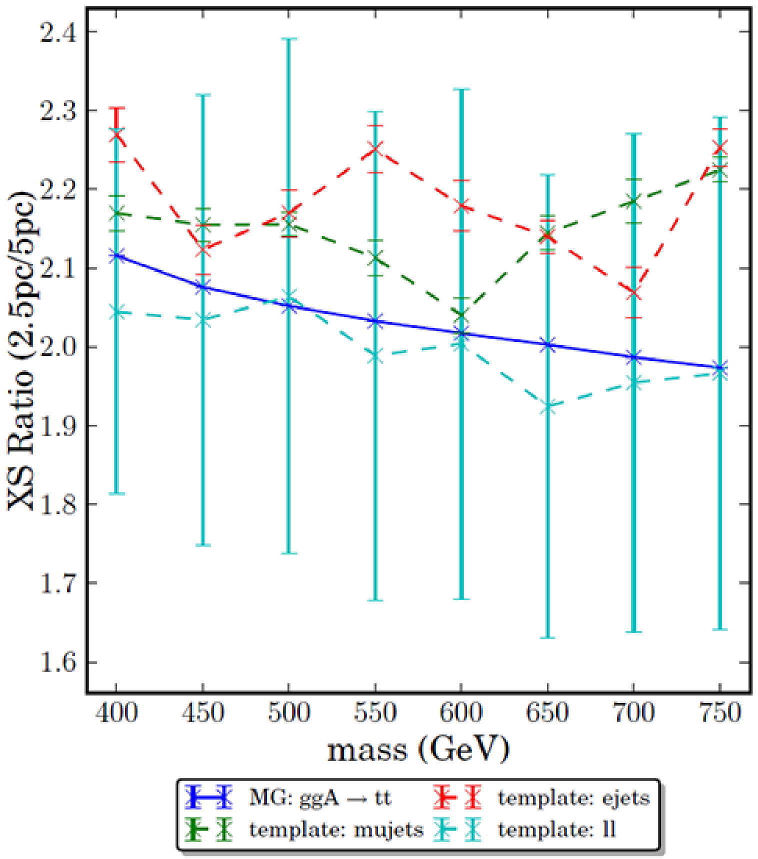
Extrapolation to 1pc width from shapes at 2.5pc width

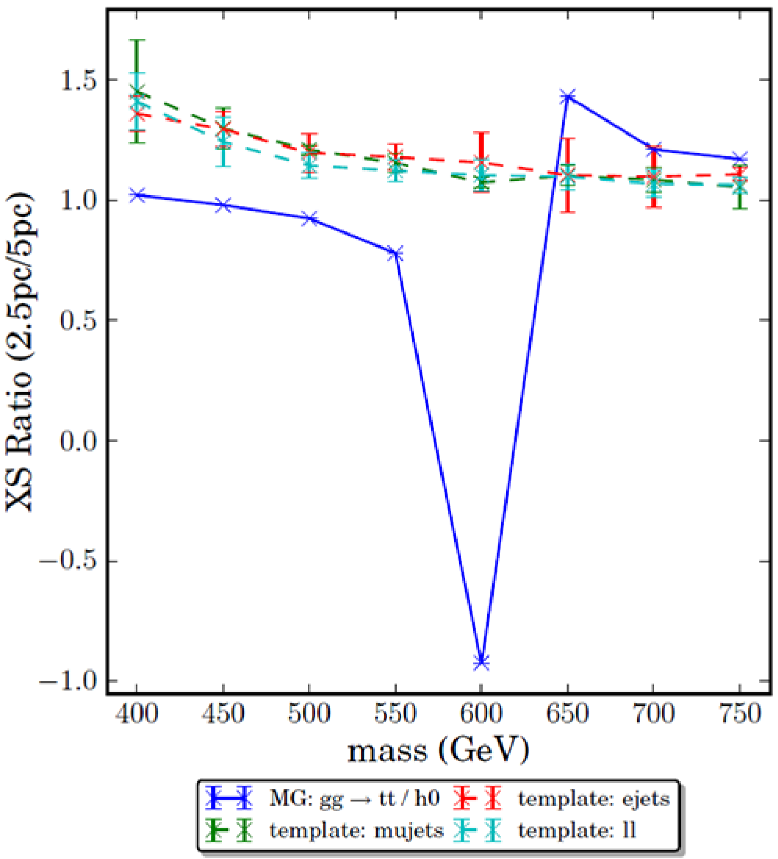
- Results for the ratio obtained
- The ratios for pseudoscalar A and scalar H are similar so we simply use the ratio for A in all cases
- The large fluctuation at 600-700 GeV mass is due to the transition between negative to positive cross-sections (negative to positive interference domination)



Extrapolation to 1pc width from shapes at 2.5pc width

- The results have been checked with the ratio at 2.5pc and 5pc width in the data samples





Statistical Methods

- The exclusion limits on the MSSM parameter space are derived from a frequentist significance test, known as the asymptotic CLs method
- We express our results as a limit on the coupling modifier, defined as the ratio of best-fit coupling to the expected SM Higgs coupling.

$$\kappa = g/g_{Htt}^{\mathrm{SM}}$$

- The sensitivity of an experiment is characterized by the median significance, using pseudo-data generated from the $\kappa = 1$(background only) hypothesis, with which one rejects values of κ incompatible with the MSSM prediction at 95% confidence level (CL).

Appendix: Statistical Methods

- Suppose the expected yield for the signal process is s_i, which may be scaled by a signal strength factor μ, and that for the background is b_i, in each bin i of the reconstructed $m_{t\bar{t}}$ spectrum

- The number of observed events n_i in the i-th bin follows the Poisson distribution:

$$\mathrm{Pois}(n_i|\mu \cdot s_i + b_i) = \frac{(\mu \cdot s_i + b_i)^{n_i}}{n_i!} e^{-(\mu \cdot s_i + b_i)}$$

- The Likelihood function, which incorporates the nuisance parameters θ is there

$$\mathcal{L}(\mathrm{data}|\mu, \theta) = \prod_{i=1}^{N} \mathrm{Pois}(n_i; \mu \cdot s_i(\theta) + b_i(\theta)) p(\theta)$$

- We define the test statistic as the profile likelihood ratio:

$$q_\mu = -2 \ln \frac{\mathcal{L}(\mathrm{data}|\mu \cdot s(\hat{\theta}_\mu) + b(\hat{\theta}_\mu))}{\mathcal{L}(\mathrm{data}|\hat{\mu} \cdot s(\hat{\theta}) + b(\hat{\theta}))}, \quad 0 \leqslant \hat{\mu} < \mu,$$

Appendix: Statistical Methods

$$q_\mu = -2\ln \frac{\mathcal{L}(\mathrm{data}|\mu \cdot s(\hat{\theta}_\mu) + b(\hat{\theta}_\mu))}{\mathcal{L}(\mathrm{data}|\hat{\mu} \cdot s(\hat{\theta}) + b(\hat{\theta}))}, \quad 0 \leqslant \hat{\mu} < \mu,$$

Here θ_μ denotes the value of that maximizes the likelihood in the numerator under the hypothesis of a signal of strength , and the denominator is the globally maximized likelihood

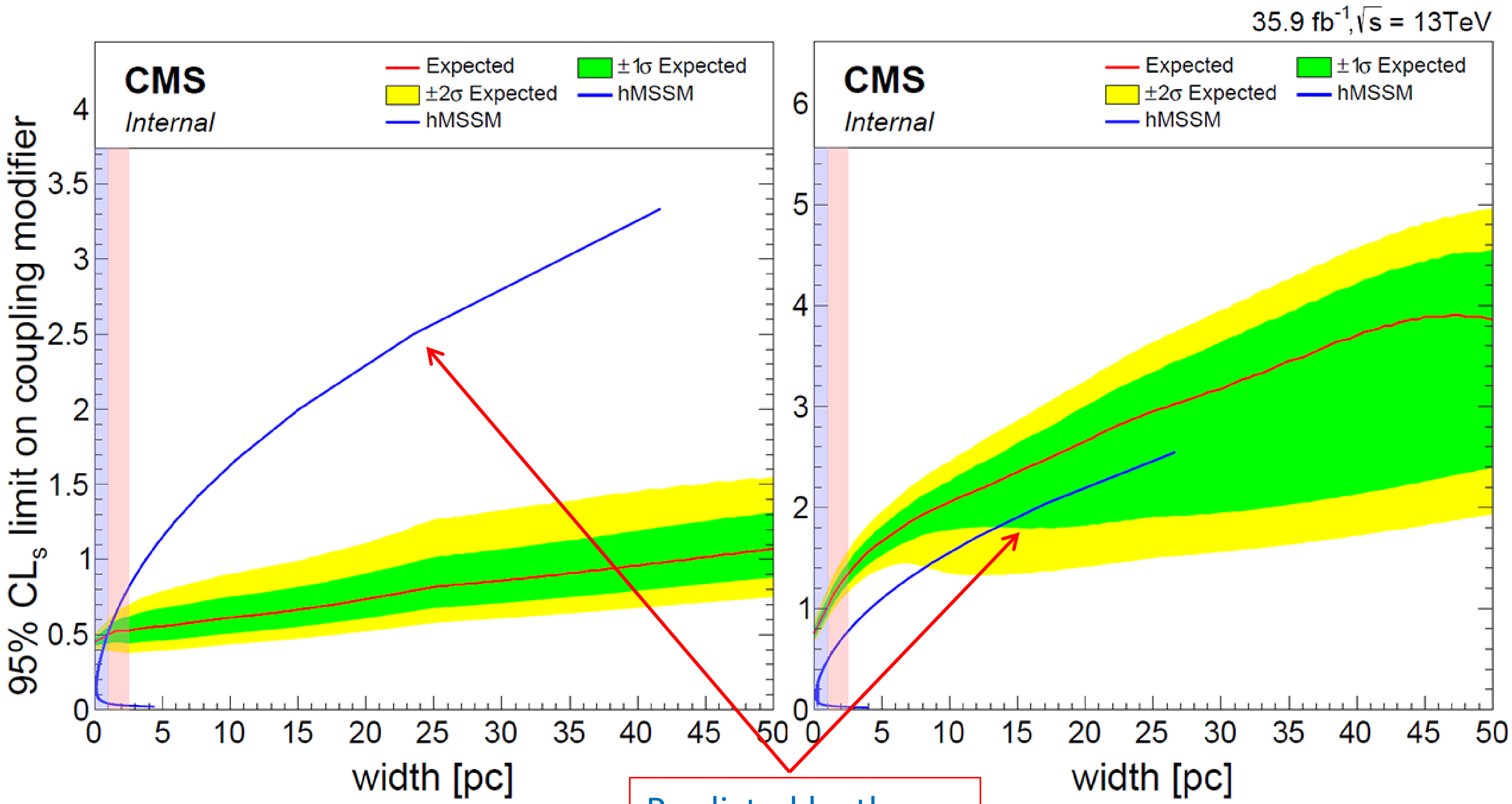
- The CLs limit is constructed based on the tail probabilities for which one would obtain a value for the test statistic q_μ larger than the observed value q_μ^{obs} for the signal + background and for the background-only hypothesis

$$\mathrm{CL_{s+b}} = P(q_\mu \geqslant q_\mu^{\mathrm{obs}}|\mu \cdot s + b),$$
$$\mathrm{CL_b} = P(q_\mu \geqslant q_\mu^{\mathrm{obs}}|b),$$

- from which we obtain the exclusion at 95% CL ($\alpha = 5\%$) by adjusting the value of μ until we reach the condition

$$\mathrm{CL_s} = \frac{\mathrm{CL_{s+b}}}{\mathrm{CL_b}} \leqslant \alpha.$$

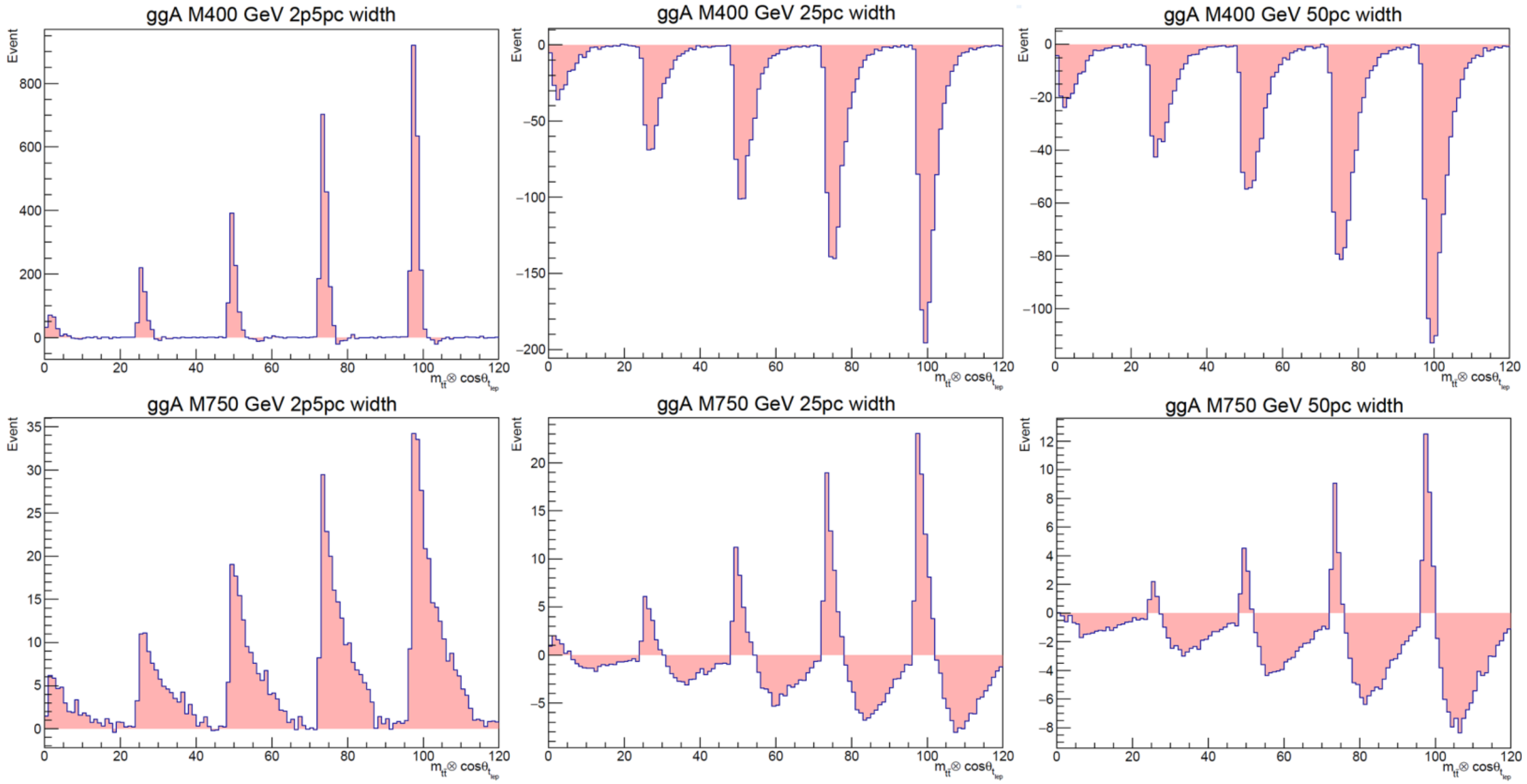
95% CL Expected limits on the Coupling Modifier



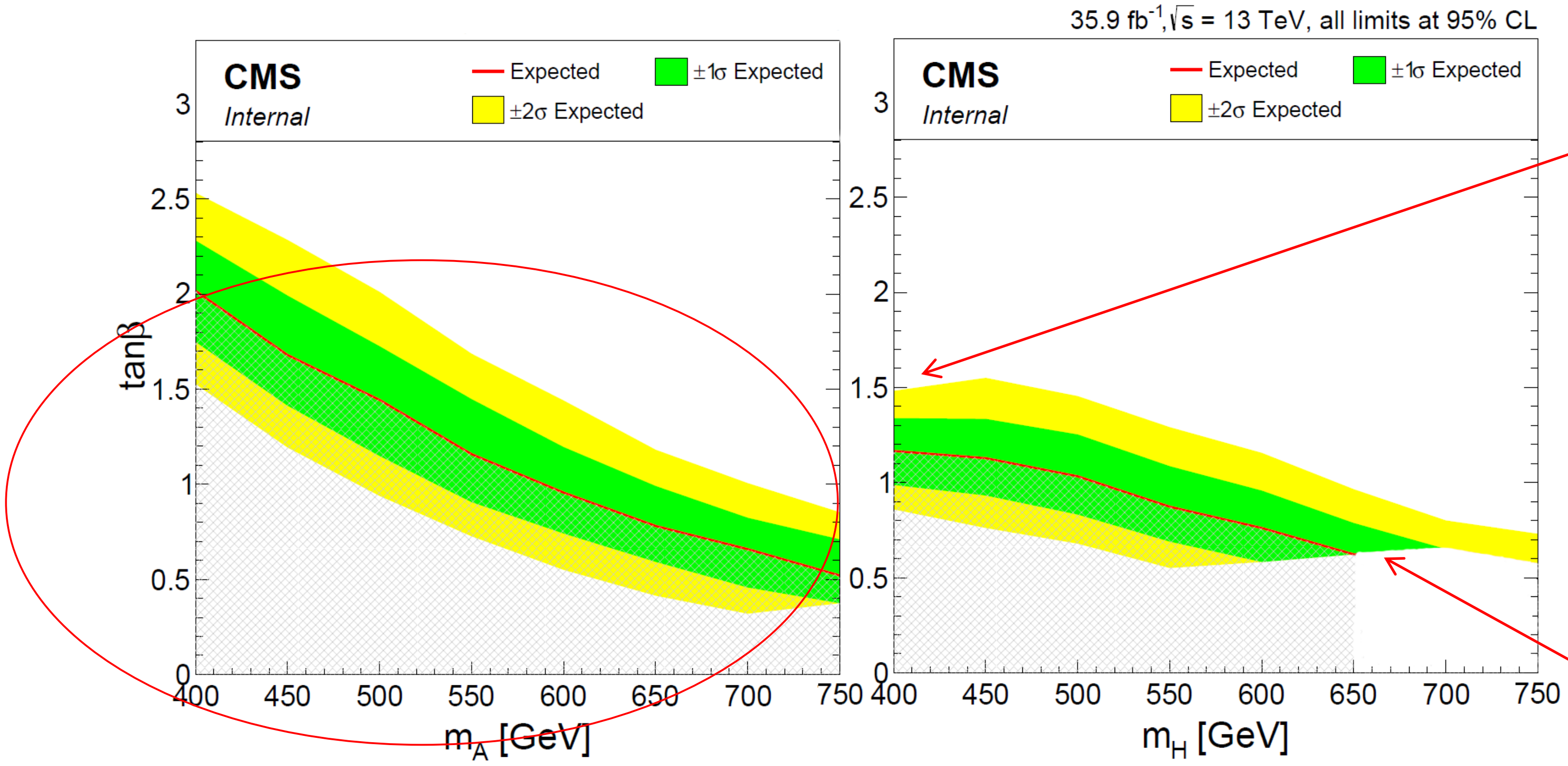
Predicted by theory

Appendix: Broadening of limit bands at higher widths and mass

- Speculated to be due to the cancelation of signal and interference contribution



Exclusion (upper limits) in $[M_A, \tan\beta]$ plane



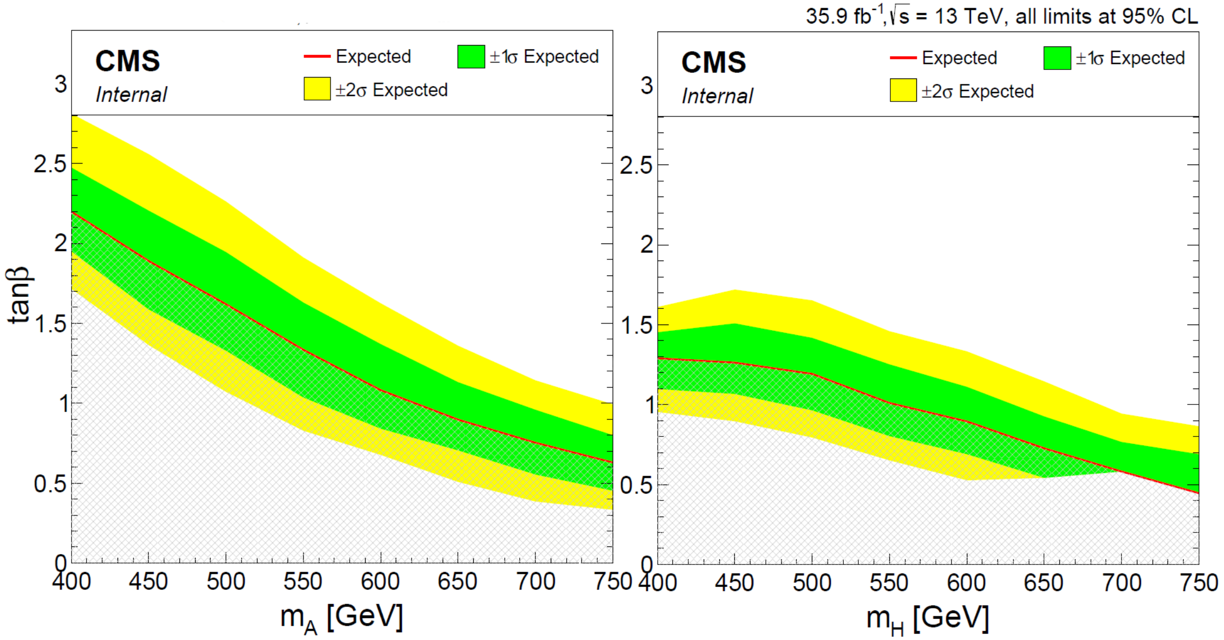
Lower sensitivity for scalar H

No intersection→ No exclusion can be made

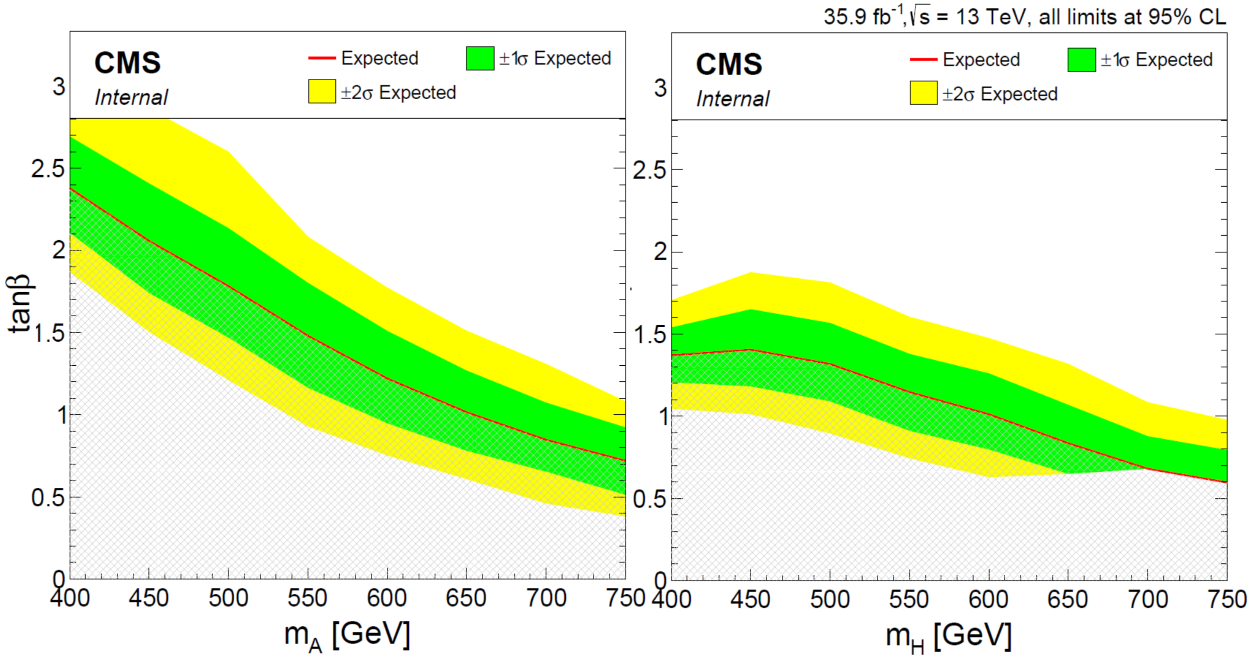
Exclude low $\tan\beta$ region

Exclusion (upper limits) in $[M_A, \tan\beta]$ plane

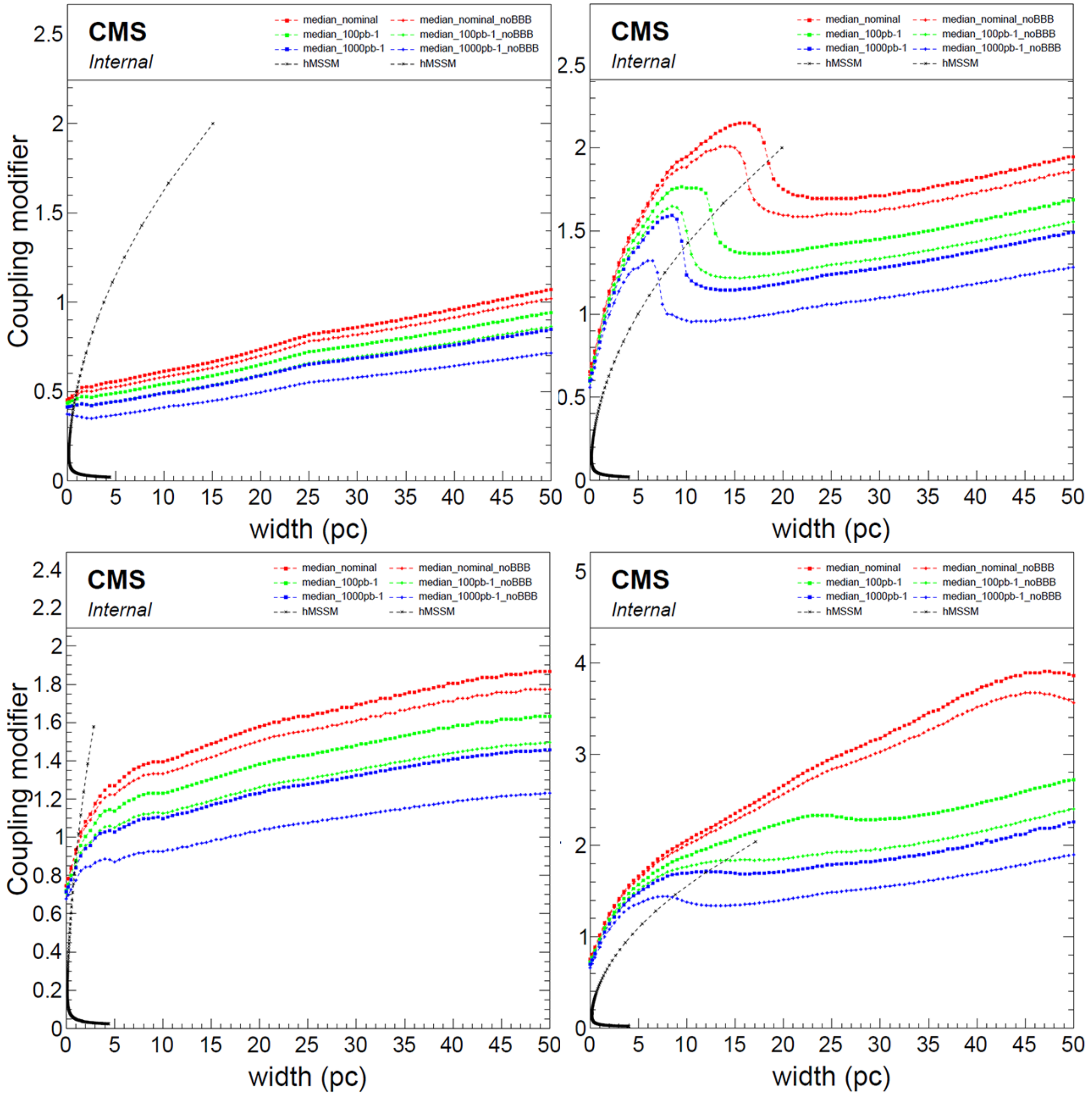
Luminosity: 100pb^{-1}



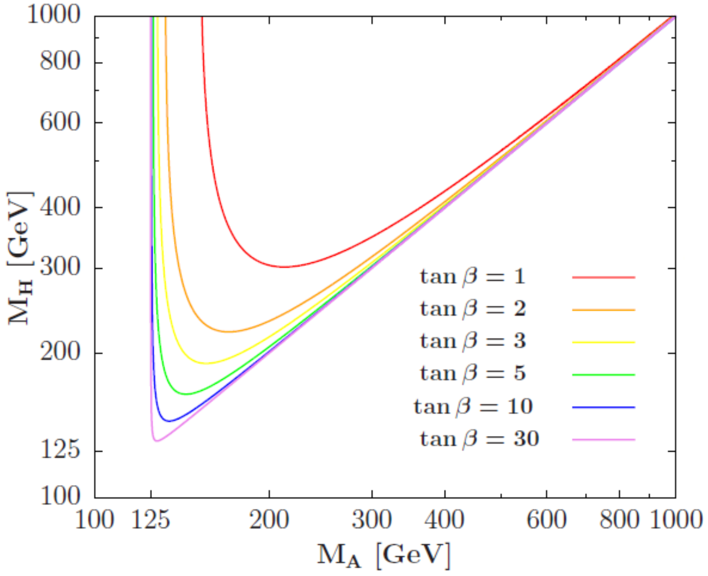
Luminosity: 1000pb^{-1}

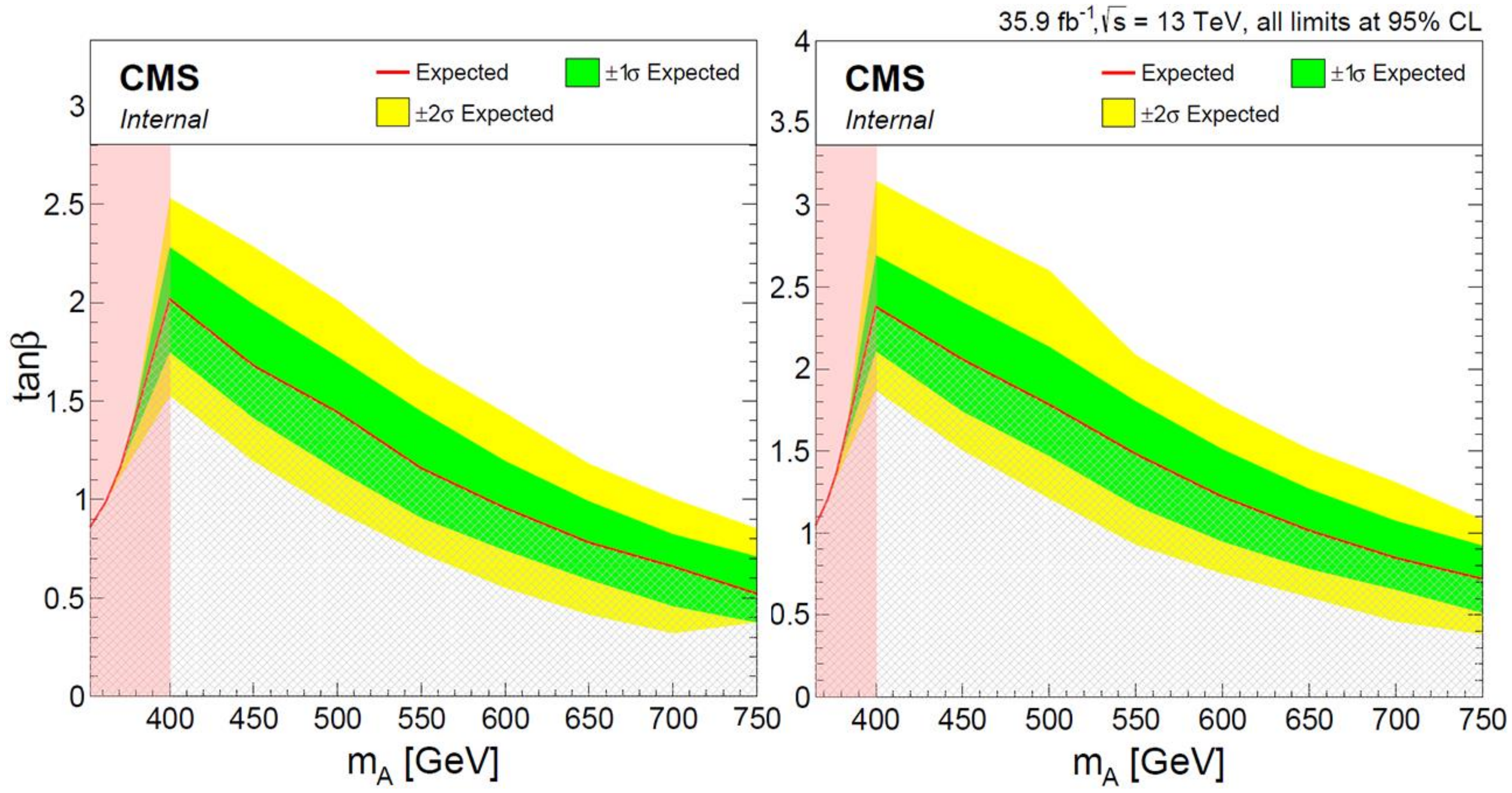


Appendix: Effect of statistical uncertainties



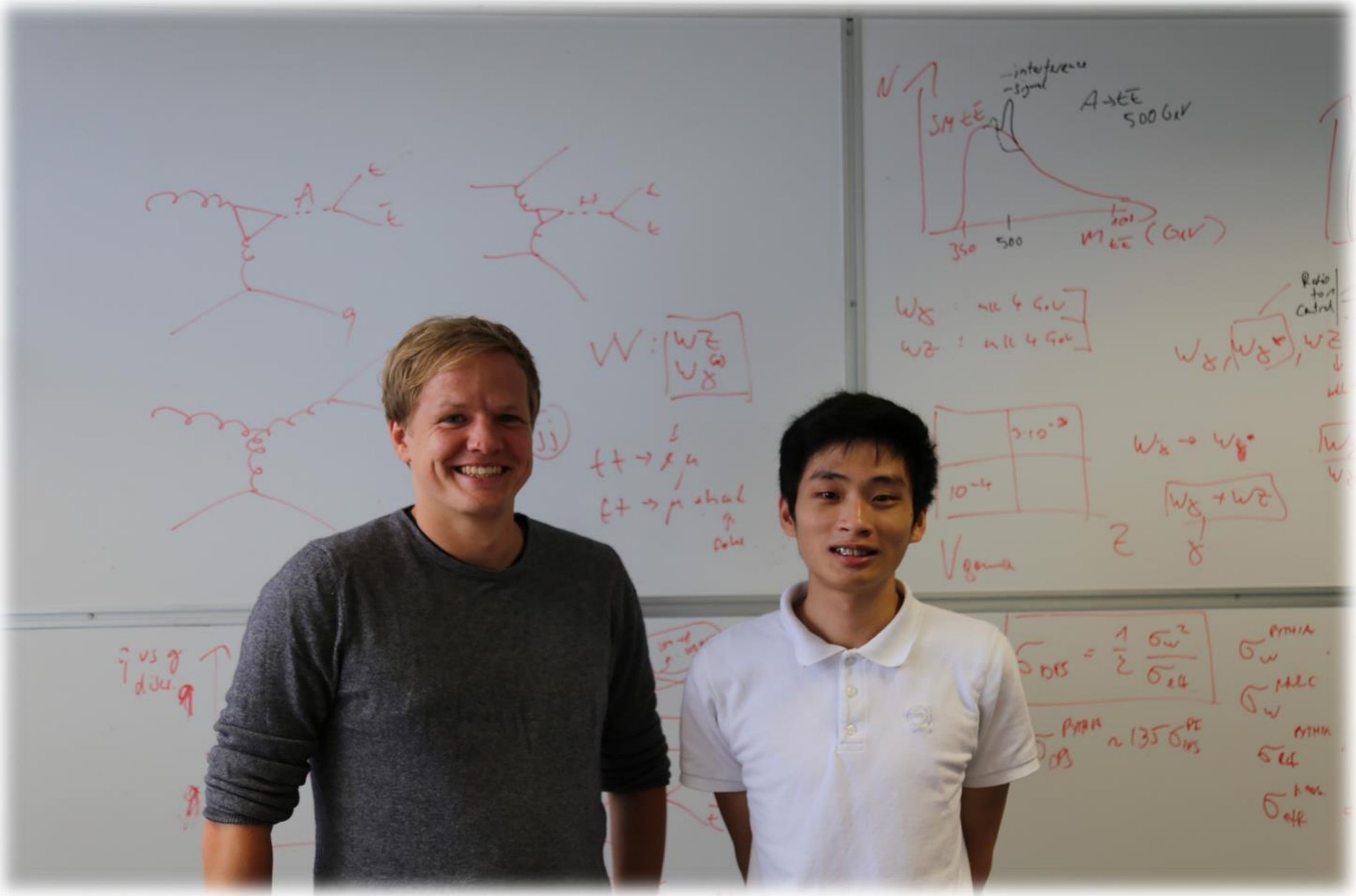
Combining the results for A and H





Acknowledgement

- Boundless gratitude to my supervisor: Jan Staggemann!



I would also like to thank Andrew Gilber, Andrey Popov, Viola Sordini, Mauro Verzetti and Muhamand Gul for valuable discussions on the MG generation of signal and interference process, on matters concerning the K-factor and on the implementation of mass and width morphing algorithms.

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Appendix: hMSSM benchmark scenario

- Radiative corrections beyond tree levels complicate the problem
- hMSSM: A particular choice of parametrization on the CP-conserving MSSM Higgs sector
- Condition in the lightest Higgs boson mass $M_h = 125$ GeV
- MSSM Higgs sector can again be characterized by 2 input parameters only
- It assumes the CP-even Higgs boson mass can be expressed in terms of

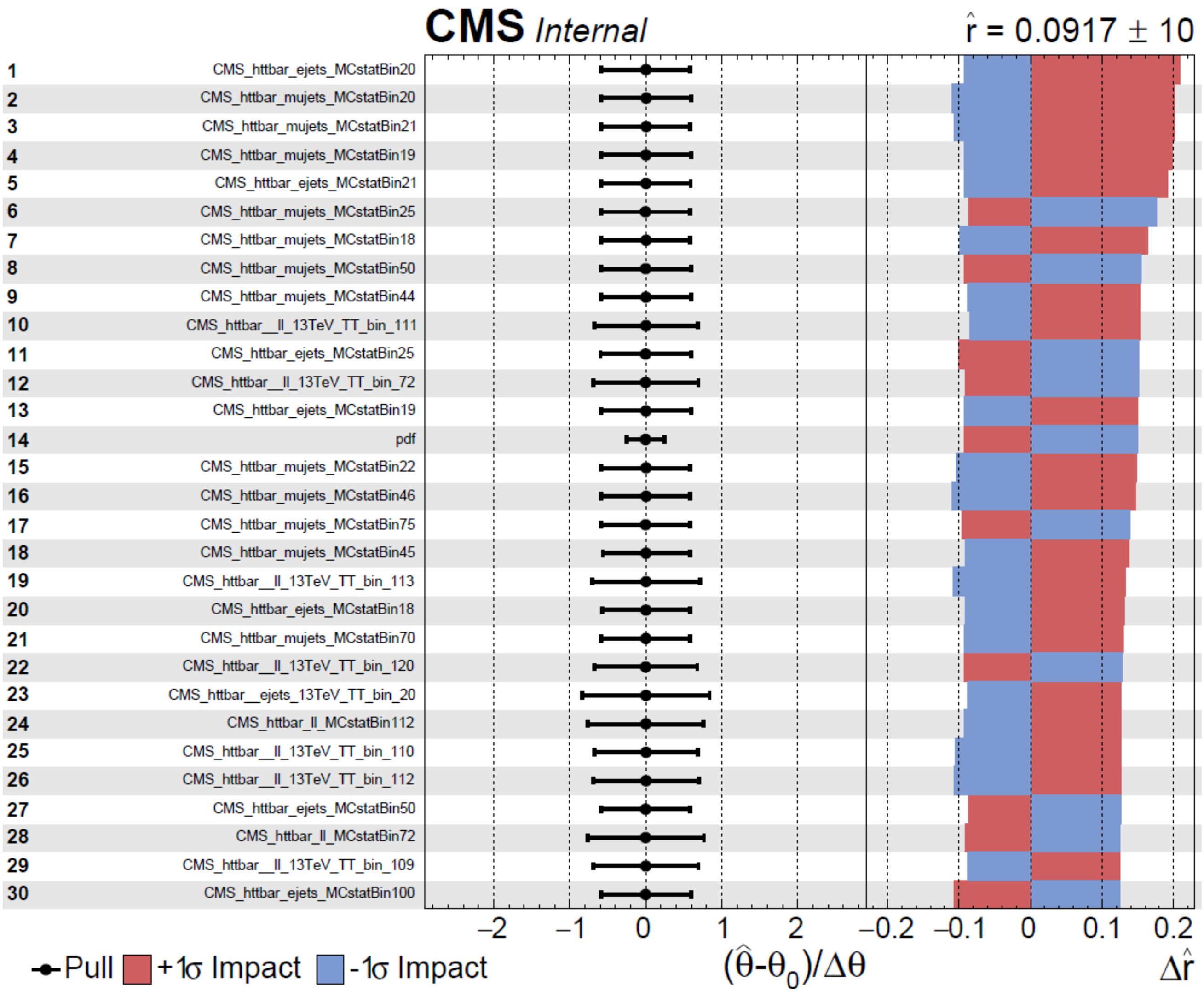
$$M_{\Phi}^2 = \begin{pmatrix} M_Z^2 \cos^2\beta + M_A^2 \sin^2\beta & -(M_Z^2 + M_A^2)\sin\beta\cos\beta \\ -(M_Z^2 + M_A^2)\sin\beta\cos\beta & M_Z^2 \sin^2\beta + M_A^2 \cos^2\beta \end{pmatrix} + \begin{pmatrix} \Delta\mathcal{M}_{11}^2 & \Delta\mathcal{M}_{12}^2 \\ \Delta\mathcal{M}_{12}^2 & \Delta\mathcal{M}_{22}^2 \end{pmatrix}$$

- The simplified expressions for M_H and α are obtained considering only the matrix element with the leading logarithmic terms for the radiative corrections, i.e. ΔM_{22}

Appendix: Systematic Uncertainties

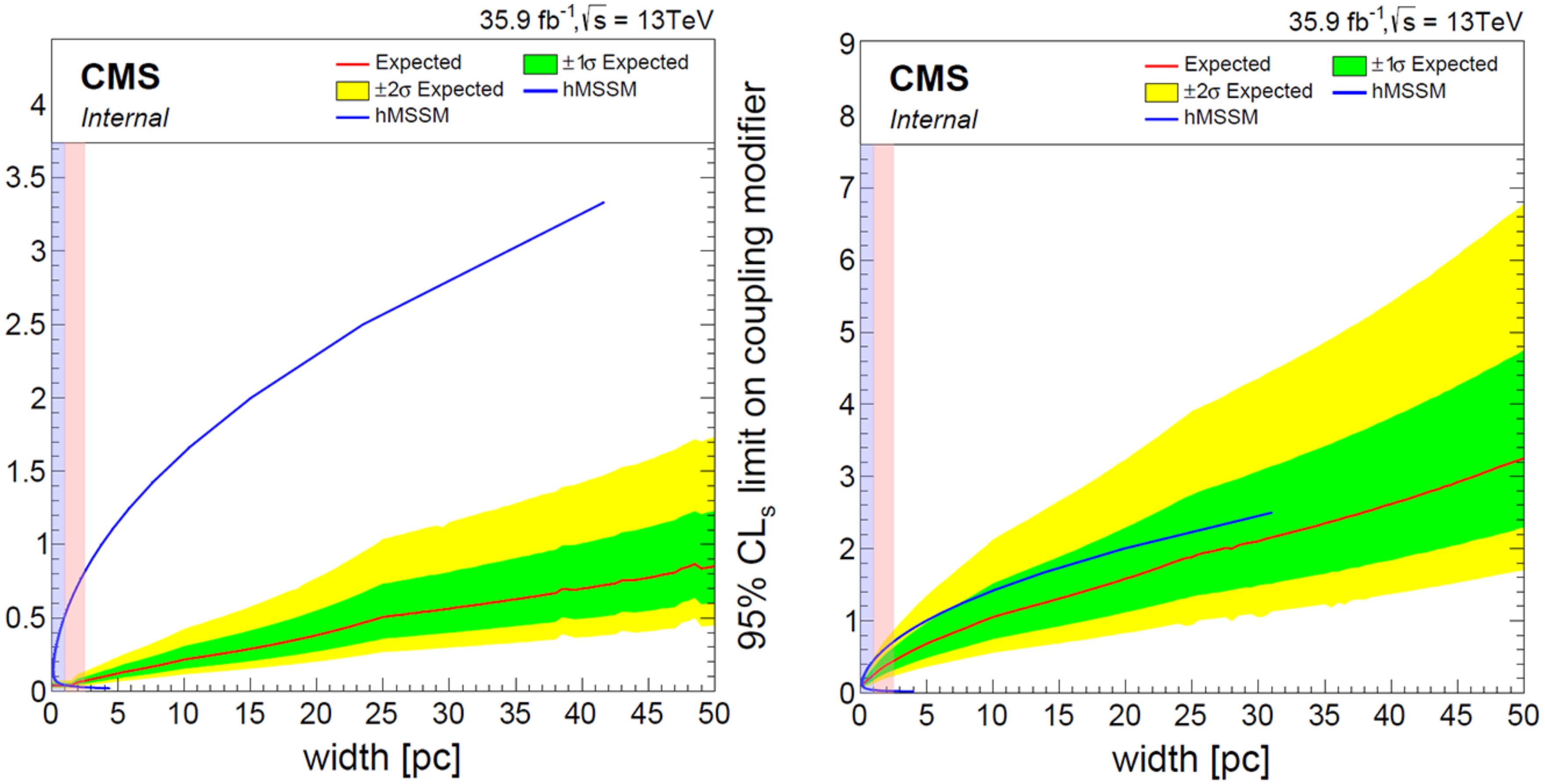
- Systematic uncertainties are handled by the means of nuisance parameters
- 797 nuisance parameters modeled with simple Gaussian constraints
- Most of these uncertainties involved are statistical in nature, i.e. due to the finite size of the simulated samples
- Nuisance parameters of this kind are collectively called bin-by-bin uncertainties

Appendix: Impacts on Nuisance Parameters



- To measure the effect of a nuisance parameter θ on a parameter of interest r (the signal strength):
  Define the impact as:
  The shift Δr that is induced when θ is fixed and moved to its +1σ or -1 σ value, with all other nuisance parameters profiled as normal.

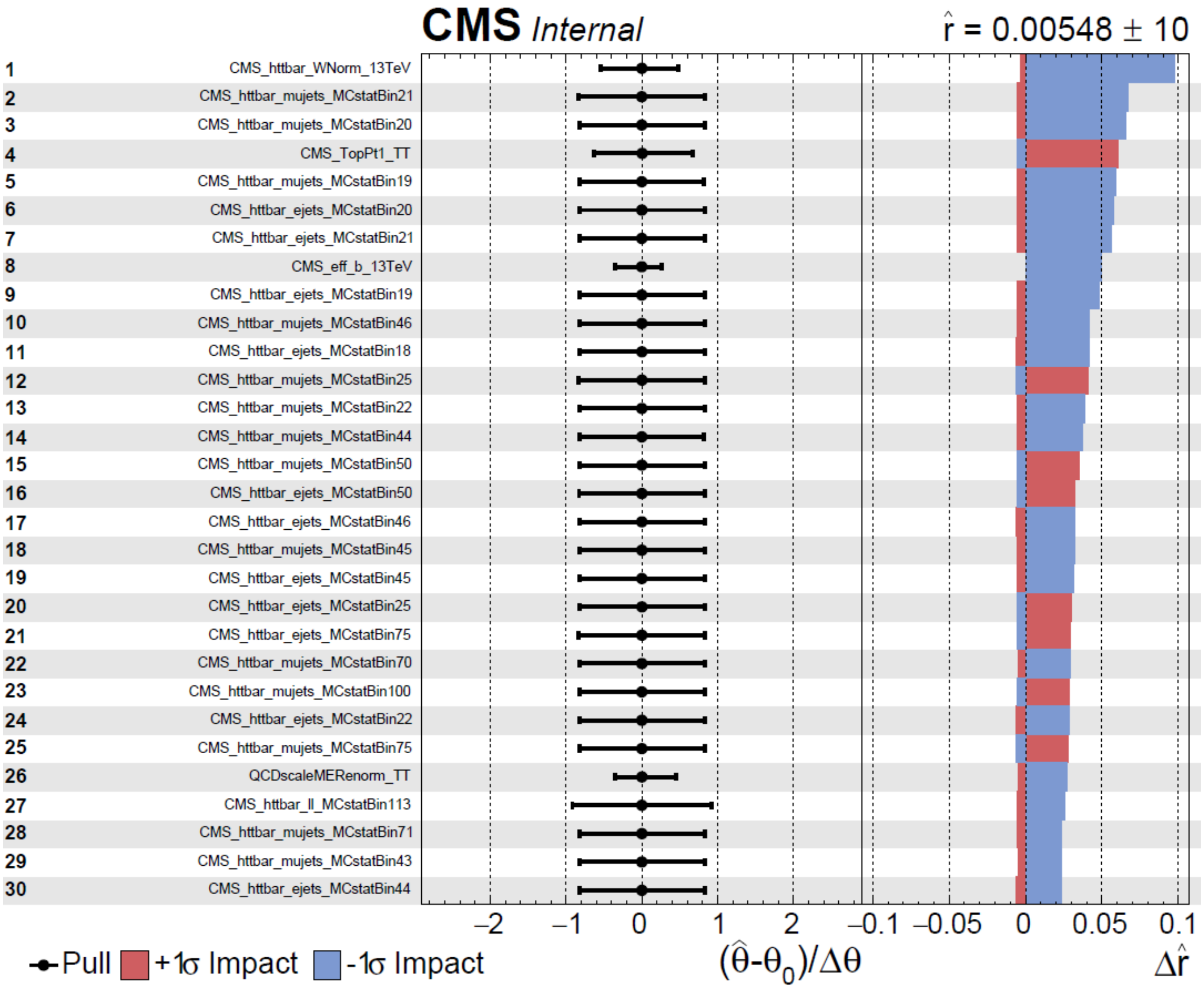
Appendix: Constraints scaling



- To reduce the effect of statistical uncertainties without recourse to simulating more data samples, we may apply a scaling of the constraints on the bin-by-bin uncertainties according to the ratio of luminosity between the extrapolated data and the nominal data

i.e. scaling the width of the Gaussian constraint on each nuisance parameter by a factor of 1/sqrt(lumisacle)

Appendix: Constraints scaling



Appendix: Formulas

Center-of-mass energy

$$s = (\tilde{p_1} + \tilde{p_2})^2$$
$$= 4E_p^2$$
$$\sqrt{s} = 2E_p = 14TeV$$

Linear vs Circular accelerator

$$E_{CM} = \sqrt{E_1} \qquad E_{CM} = E_1 + E_2$$

Yukawa coupling to top quarks

$$\mathcal{L} \supset \frac{m_t}{v}\left(g_{Htt}\, t\bar{t} + i g_{Att}\, t\gamma_5 \bar{t}\right)\Phi$$

CP-even Higgs obtained from the rotation of Higgs doublet filed by the mixing angle α

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}$$

Appendix: Concepts

- Pile-up: number of proton collisions per bunch crossing (How many interactions we can expect to see when we record an event)
- Asimov data set: the one data set in which all observed quantities are set equal to their expected values
- Wilk's theorem: The profile likelihood ratio $-2log\lambda$ distributes asymptotically as χ^2, when the null hypothesis is true
- Wald's theorem: generalizes Wilk's theorem to non-null hypothesis: non-central χ^2 $$q_\mu = \begin{cases} \frac{(\mu-\hat{\mu})^2}{\sigma^2} & \hat{\mu} < \mu, \\ 0 & \hat{\mu} > \mu, \end{cases}$$
- Pseudorapidity describes the angle of a particle relative to the beam axis:

$$\eta = -\ln\left[\tan(\theta/2)\right] \qquad \text{where } \theta \text{ is the polar angle}$$

- How many boson associated with a particular force: EM - U(1) = 1**2 = 1; Weak - SU(2) = 2**2 - 1 (Special Group) = 3; Strong - SU(3) = 3**2 - 1 = 8
- Pseudoscalar particles are particles with spin 0 (scalar) and odd parity (pseudo):
  A particle with no intrinsic spin with wave function that changes sign under parity inversion

Appendix: Pile-up