Massive Scalar Polarizations of Gravitational Waves

Background Knowledge: Polarizations

- Travelling oscillations of gravitational field.
- Gravitational wave polarizations can be thought of as components of gravitational waves.
- Green: Tensor
- Red: Vector
- Black: Scalar
- (Sathyaprakash and Isi)

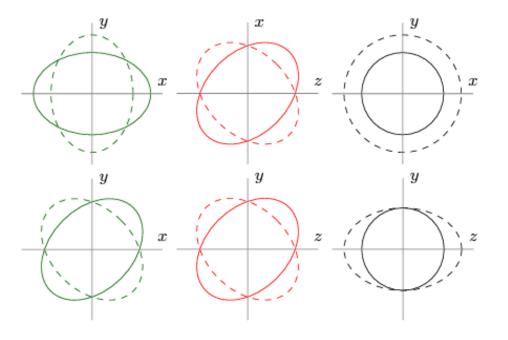


Figure Source: Isi

Background Knowledge: Polarizations

- GR predicts massless gravitational waves and rejects non-tensor polarizations.
- There can also be scalar polarizations of gravitational waves, which is massive, in other theories.
- Massless: graviton has no mass, travels in the speed of light
- Massive: graviton has a mass, travels in a speed slower than the speed of light and depends on frequency.
- (Isi, Will and Yang)

Background Knowledge: Polarizations

Call the hypothesis having both massive scalar and massless tensor polarizations as the scalar-tensor hypothesis (as opposed to GR).

Background knowledge: Polarizations

Speed of massive scalar polarizations are given by,

$$\frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{h^2 f^2}$$

- Where m_g and f is the rest mass and the frequency of the graviton, respectively.
- The higher the frequency, the faster the massive scalar polarization (dispersion).
- The different traveling speed causes a time delay.
- (<u>Will</u> and <u>Yang</u>)

Aim

- To test how likely there are scalar polarizations supported by existing data.
- By looking at the time delay between the massless tensor polarizations and massive scalar polarizations of gravitational wave signals from binary black holes (BBHs).
- $\blacktriangleright P(H_{ST} \mid \{\Delta t_i\})$
- > $\{\Delta t_i\}$ is the set of time differences between signal arrivals.

Things to try suggested by supervisor

- $\blacktriangleright P(H_{ST} \mid \{\Delta t_i\}) \propto P(\{\Delta t_i\} \mid H_{ST}) P(H_{ST})$
- Obtain samples of time delays of twin signals Δt_{TS} . (Tensor then Scalar)
- A twin signal is the pair of tensor signal and scalar signal from the same source.
- With samples of Δt_{TS} , by simulating time of arrivals of tensor signals, signals of tensor scalar polarizations can be obtained, thus getting $P(\{\Delta t_i\} \mid H_{ST})$

Time Delay Samples

- ▶ To get time delay samples, a population of BBHs may be simulated.
- As Δt_{TS} is given by $\Delta t_{TS}(m_g, f, D_L)$, keeping m_g constant, the only parameters of BBHs required to be simulated is the luminosity distance D_L and the total masses ($M = m_1 + m_2, m_1 \ge m_2$) of the BBHs.
- The total masses of BBHs are required because the frequency of GWs are changing w.r.t. time, so the f_{isco} is chosen to represent the signal.
- The goal is P(M) and $P(D_L)$ so that we can have samples of M and D_L for samples of Δt_{TS} .

Time Delay Samples (con't)

> The dependence of f_{isco} with the total mass M is,

•
$$f_{isco} = 4.4(\frac{M_{\odot}}{M})$$
kHz

- ▶ We need the probability density distribution of m_1 , m_2 and D_L to get samples of them and hence samples of time delay Δt_{TS} .
- ► (<u>Veitch</u>)

Time Delay Samples: $P(m_1, m_2)$

$$P(m_{1}) = \left(\frac{1-\alpha}{m_{max}^{1-\alpha} - m_{min}^{1-\alpha}}\right) m_{1}^{-\alpha}$$

$$P(m_{2}|m_{1}) = \left(\frac{1+\beta_{q}}{m_{1}^{1+\beta}q - m_{min}^{1+\beta}q}\right) m_{1}^{\beta_{q}}q^{\beta_{q}}$$

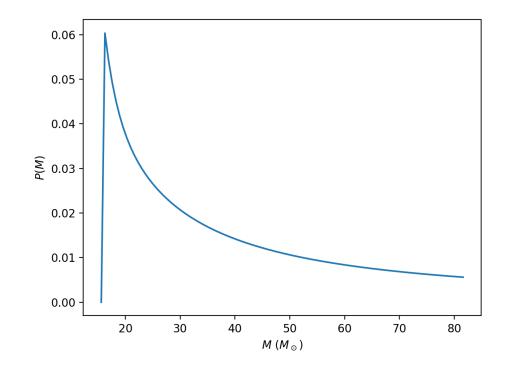
$$P(m_{1}, m_{2}) = \left(\frac{1-\alpha}{m_{max}^{1-\alpha} - m_{min}^{1-\alpha}}\right) \left(\frac{1+\beta_{q}}{m_{1}^{1+\beta}q - m_{min}^{1+\beta}q}\right) m_{1}^{-\alpha+\beta_{q}}q^{\beta_{q}}$$

> The primary mass and secondary mass (when given a primary mass) both follows a power law.

- The appearance of the mass ratio $q = \frac{m_2}{m_1}$ in the distribution creates the tendency for the primary mass to be similar in magnitude to the secondary mass when β_q is positive (0 < q < 1).
- (Abbott)

Time Delay Samples: P(M)

- P(M) can be obtained by changing variable from (m_1, m_2) to (m_1, M) and then marginalizing over m_1 .
- Doing the marginalization numerically, P(M)



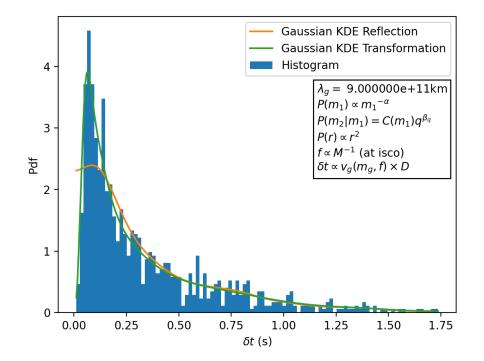
Time Delay Samples: $P(D_L)$

By the cosmological principle, the number of BBHs and hence the probability of having a BBH at luminosity distance D_L can be assumed to be proportional to the volume of a thin shell of the sphere at the distance

►
$$P(D_L) = \frac{3D_L^2}{D_{L_{max}}^3 - D_{L_{min}}^3}$$

Time Delay Samples

► By getting samples of M and D_L from their respective PDFs, samples of Δt_{TS} is obtained, and a Gaussian KDE is used to estimate $P(\Delta t_{TS})$.



Time Difference between All Signal Arrivals

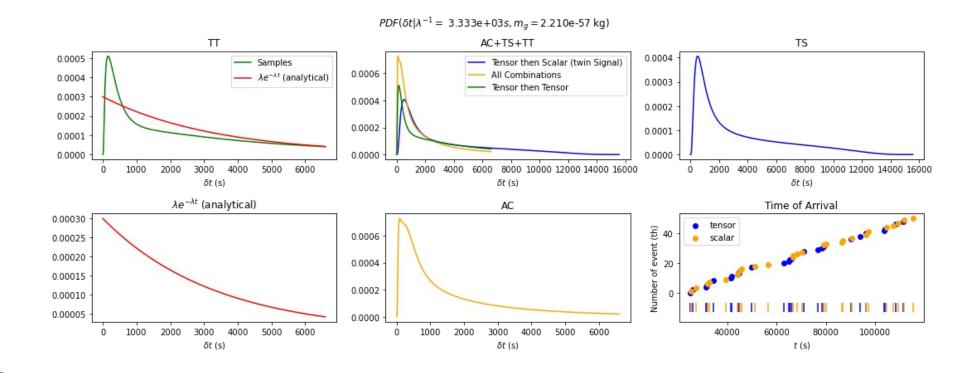
- Define a time of detection.
- Generating tensor signals, then adding their corresponding twin scalar signals, and with scalar signals coming from earlier tensor signals, the signal arrival times with both polarizations can be simulated.
- How early in time tensor signals are generated equals to the maximum time delay (beyond the maximum time delay scalar twin signal do not arrive).

Time Difference between All Signal Arrivals

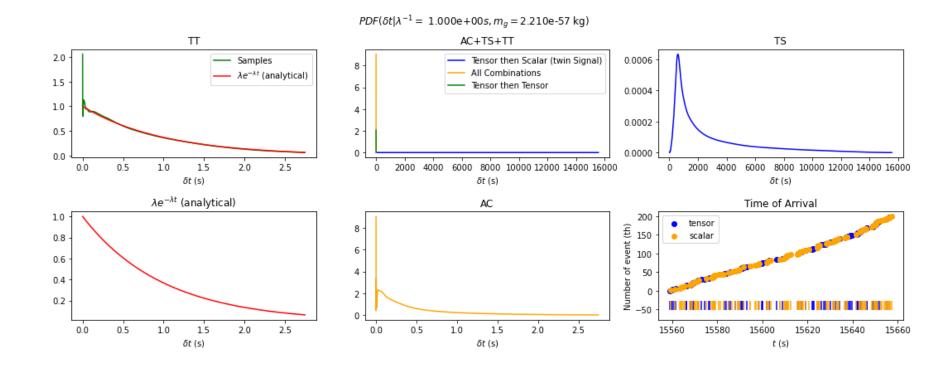
- Tensor signals have to be first generated.
- Tensor signal arrival times can be modelled as a homogenous Poisson process,

$$\blacktriangleright P(n|\lambda) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

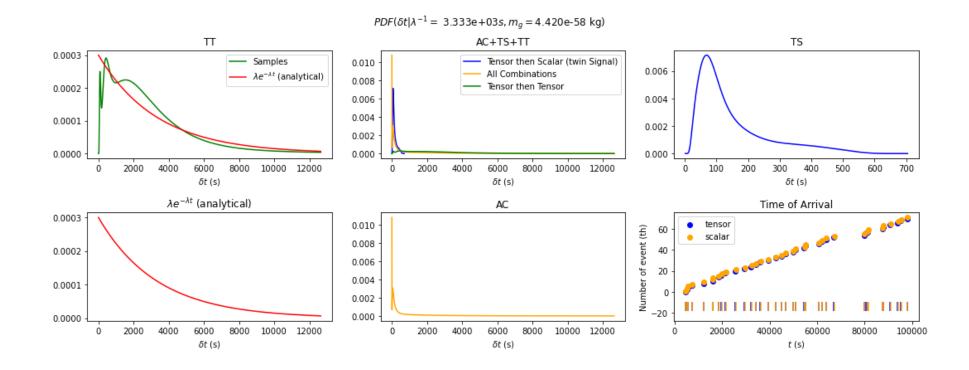
Because of the independence between events (and as the probability of receiving a signal scales with time).



Time Difference: TT~TS



Time Difference: TT<TS



Time Difference: TT>TS

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