Emulating Cosmological Growth Functions with Machine Learning

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Background

- Particle mesh simulation
- Evolves dark matter under gravity
- Updates displacement, momentum and force

$$x(a_1) = x(a_0) + \frac{H_0}{a_r H(a_r)} \frac{D(a_1) - D(a_0)}{dD/da|_{a_r}} p(a_r)$$

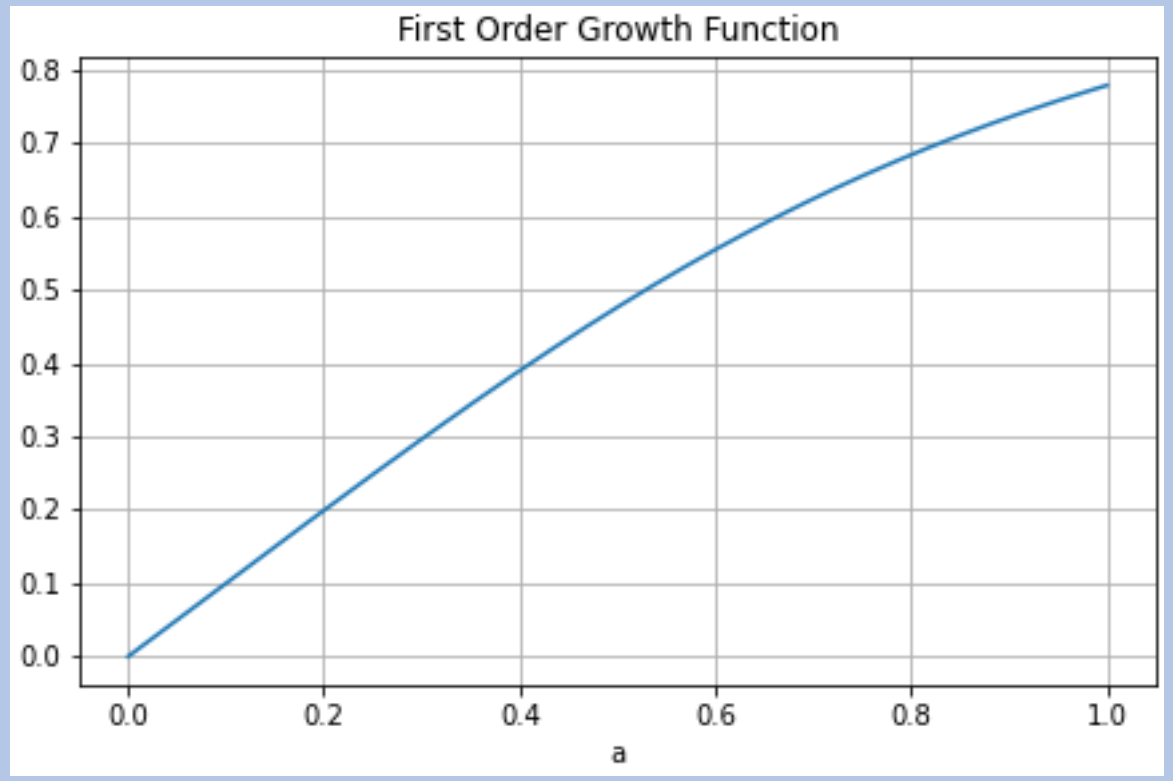
Cosmological Growth Function

- Evolution of density field
- Large scale clustering
- Solving an ODE in every step
- Time consuming!

$$a^2 \frac{d^2 D_1(a)}{da^2} + \left(\Omega_\Lambda(a) + \frac{\Omega_m(a)}{2} + 2 \right) a \frac{dD_1(a)}{da} = \frac{3}{2} \Omega_m(a) D_1(a)$$

Cosmological Growth Function

- Function of Ω_m and a
- 2 orders and 3 time derivatives each

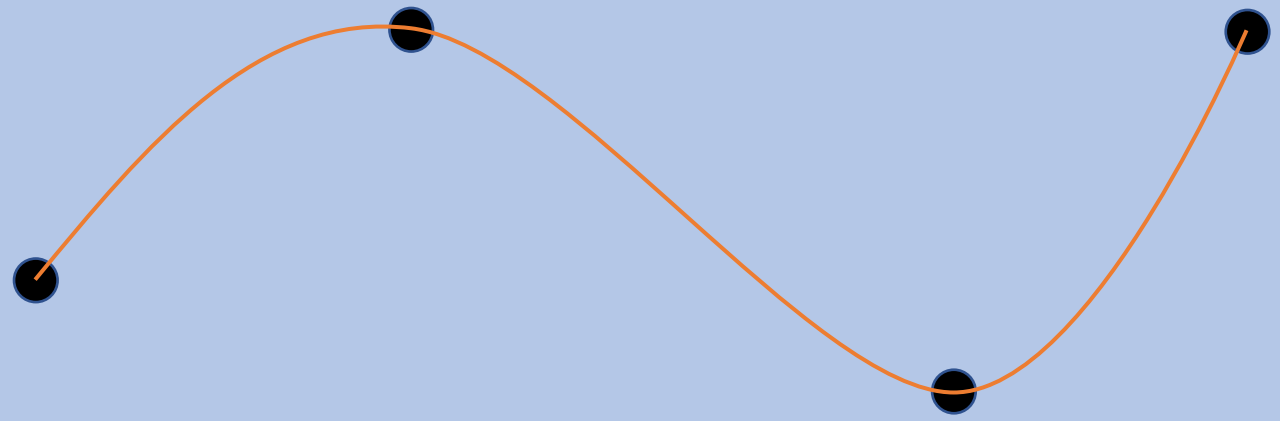


Method

- Emulator
- Artificial neural network + B-spline interpolation
- Input cosmological parameters (Ω_m)
- Interpolate over a

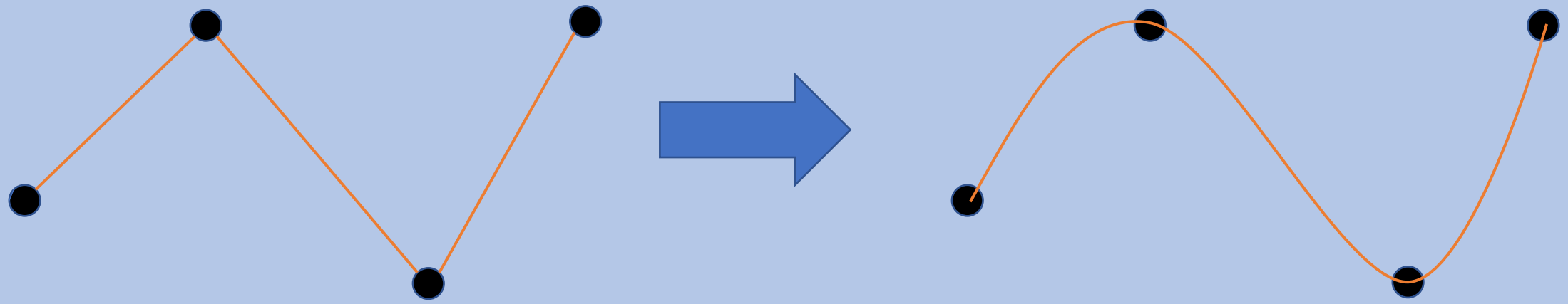
B-spline Interpolation

- Interpolation



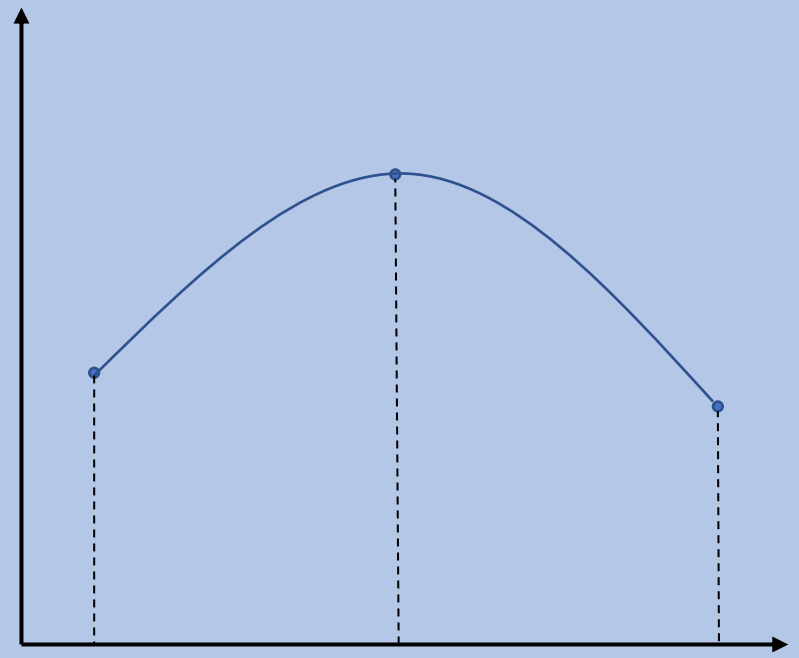
B-spline Interpolation

- “Basis spline”
- High order spline constructed by low order splines
- Interpolate based on knot positions and values

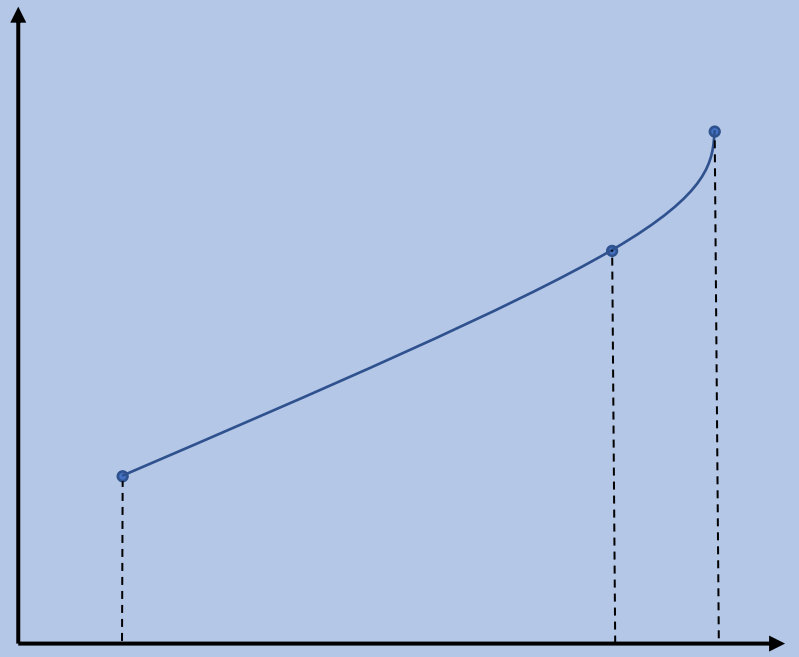


Artificial Neural Network

- Predict the knot positions and values

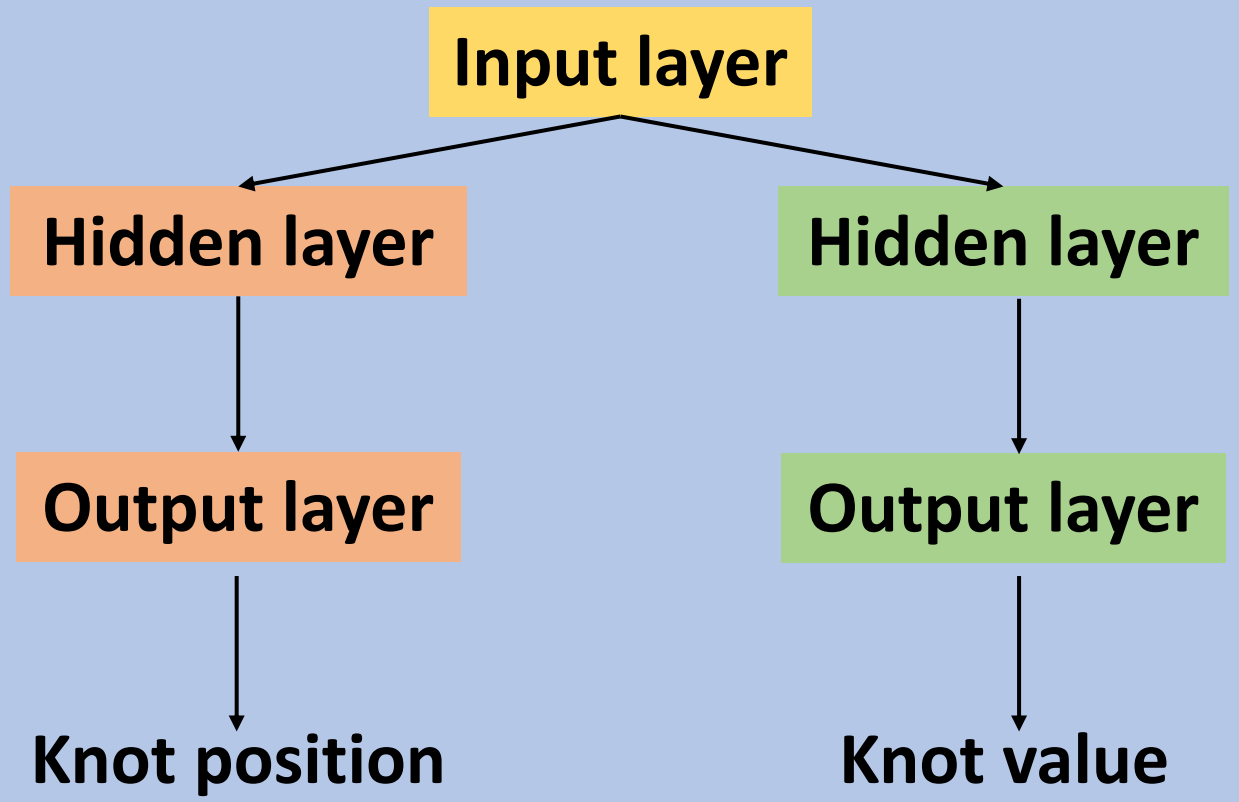


or



Artificial Neural Network

- Structure



Emulator Settings

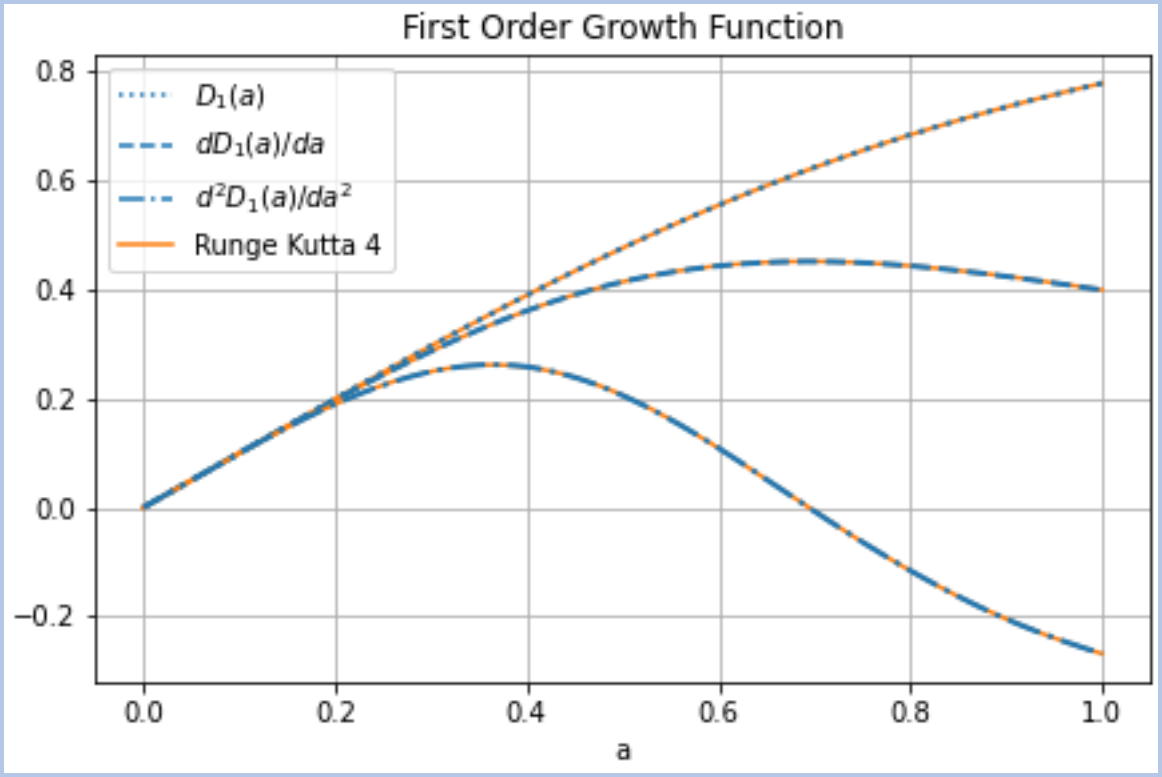
- Input of : Ω_m and a
- 1 hidden layer for both part
- 64 neurons each in input layer and hidden layers
- 8 knots with one fixed at $a = 0$
- Third order b-spline interpolation

Advantages

- Smooth, differentiable function
- Simple NN structure

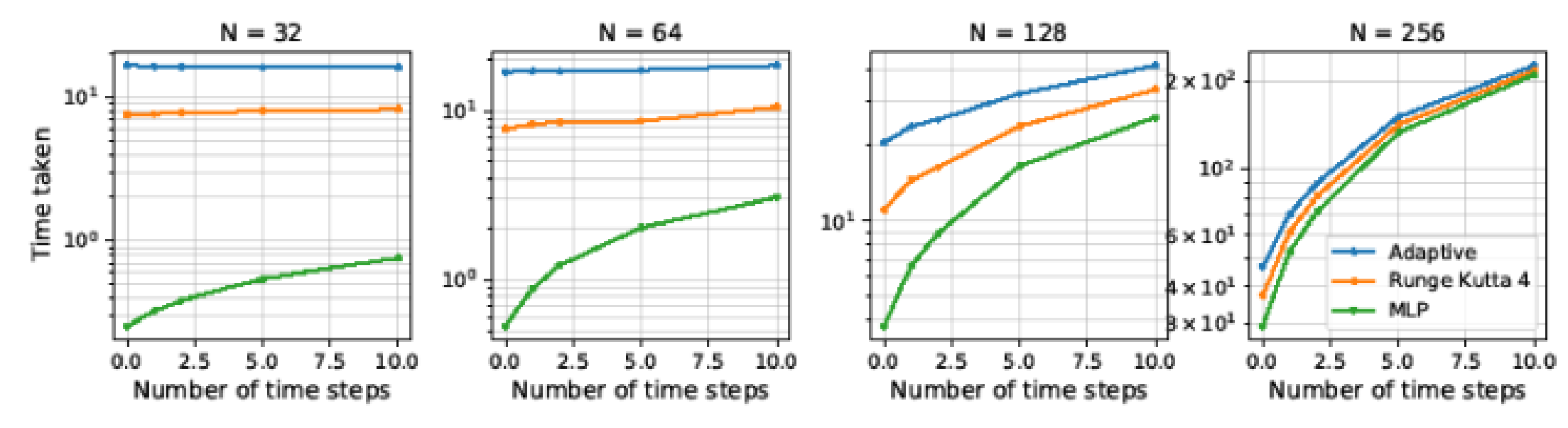
Performance

- Accuracy



Performance

- Efficiency



Future Work

- Expand the input cosmological parameters
- $w_0, w_a, \Omega_{\mathrm{K}}$

Q & A

Thank you

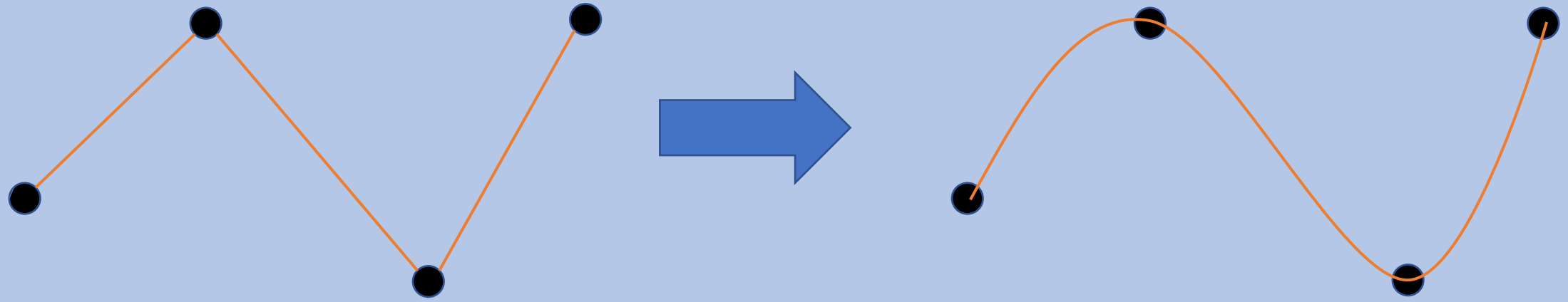
Growth Function

- Second order growth function:

$$a^2 \frac{d^2 D_2(a)}{da^2} + \left(\Omega_\Lambda(a) + \frac{\Omega_m(a)}{2} + 2 \right) a \frac{dD_2(a)}{da} = \frac{3}{2} \Omega_m(a) \left[D_2(a) - \left(D_1^+(a) \right)^2 \right]$$

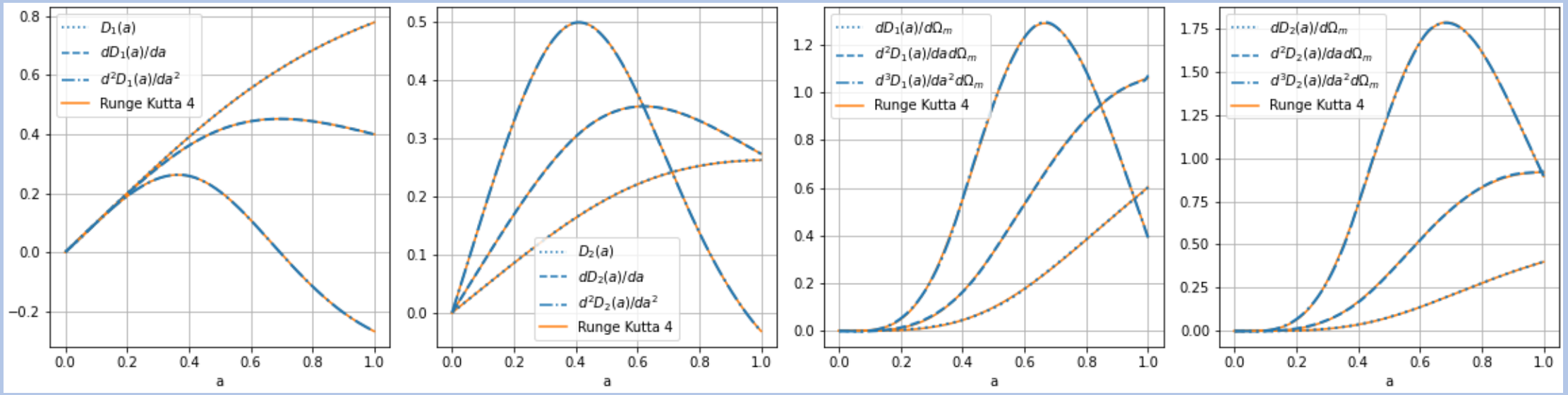
B-spline Interpolation

- $B_{i,k+1} = \frac{x-t_i}{t_{i+1}-t_i} B_{i,k} + \frac{t_{i+1}-x}{t_{i+1}-t_i} B_{i+1,k}$



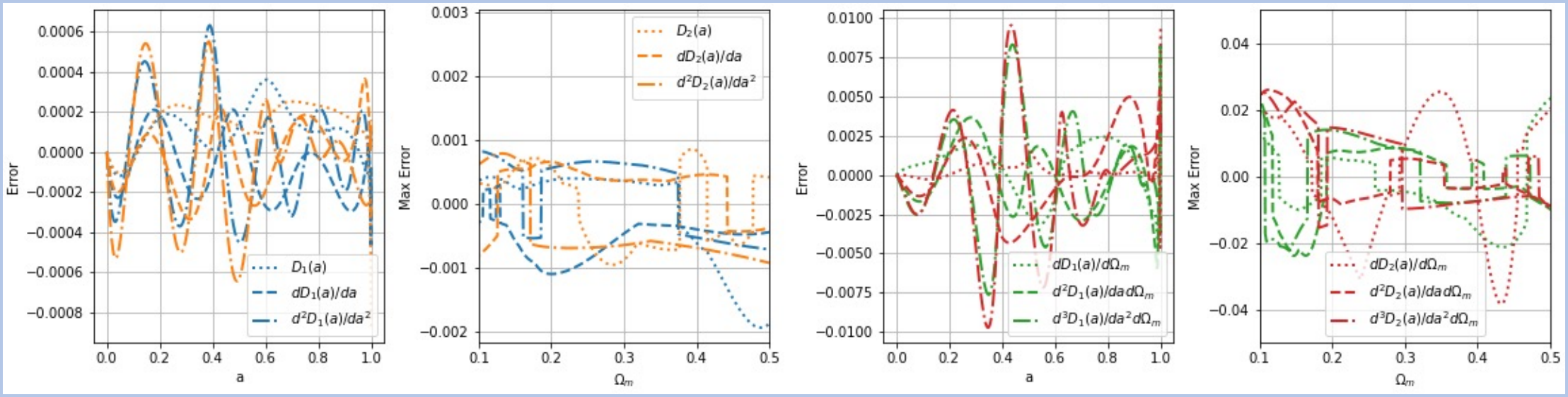
Performance

- Accuracy



Performance

- Accuracy



Performance

- Accuracy