Interaction of photons with gravitational waves

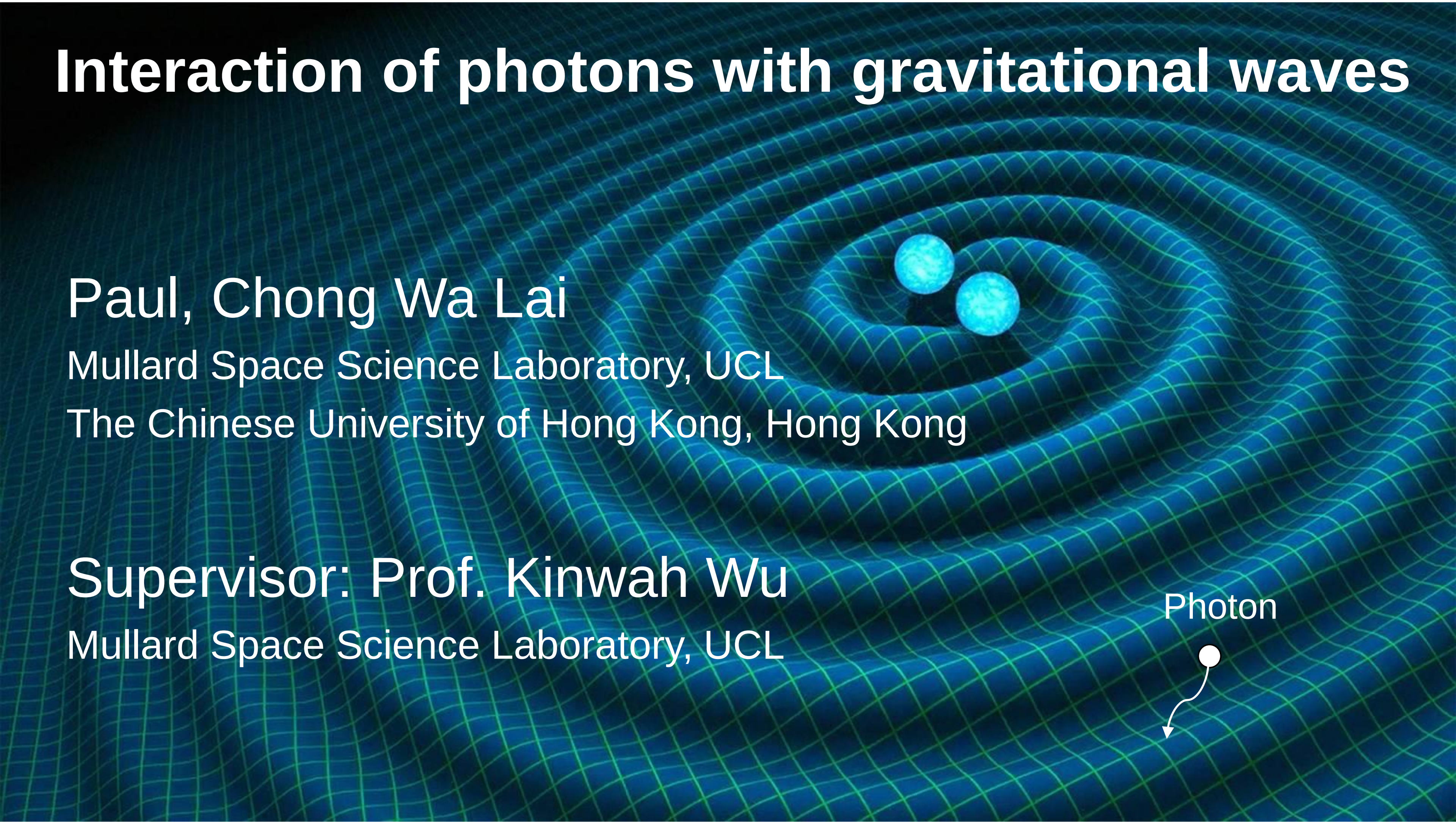
Paul, Chong Wa Lai

Mullard Space Science Laboratory, UCL

The Chinese University of Hong Kong, Hong Kong

Supervisor: Prof. Kinwah Wu

Mullard Space Science Laboratory, UCL



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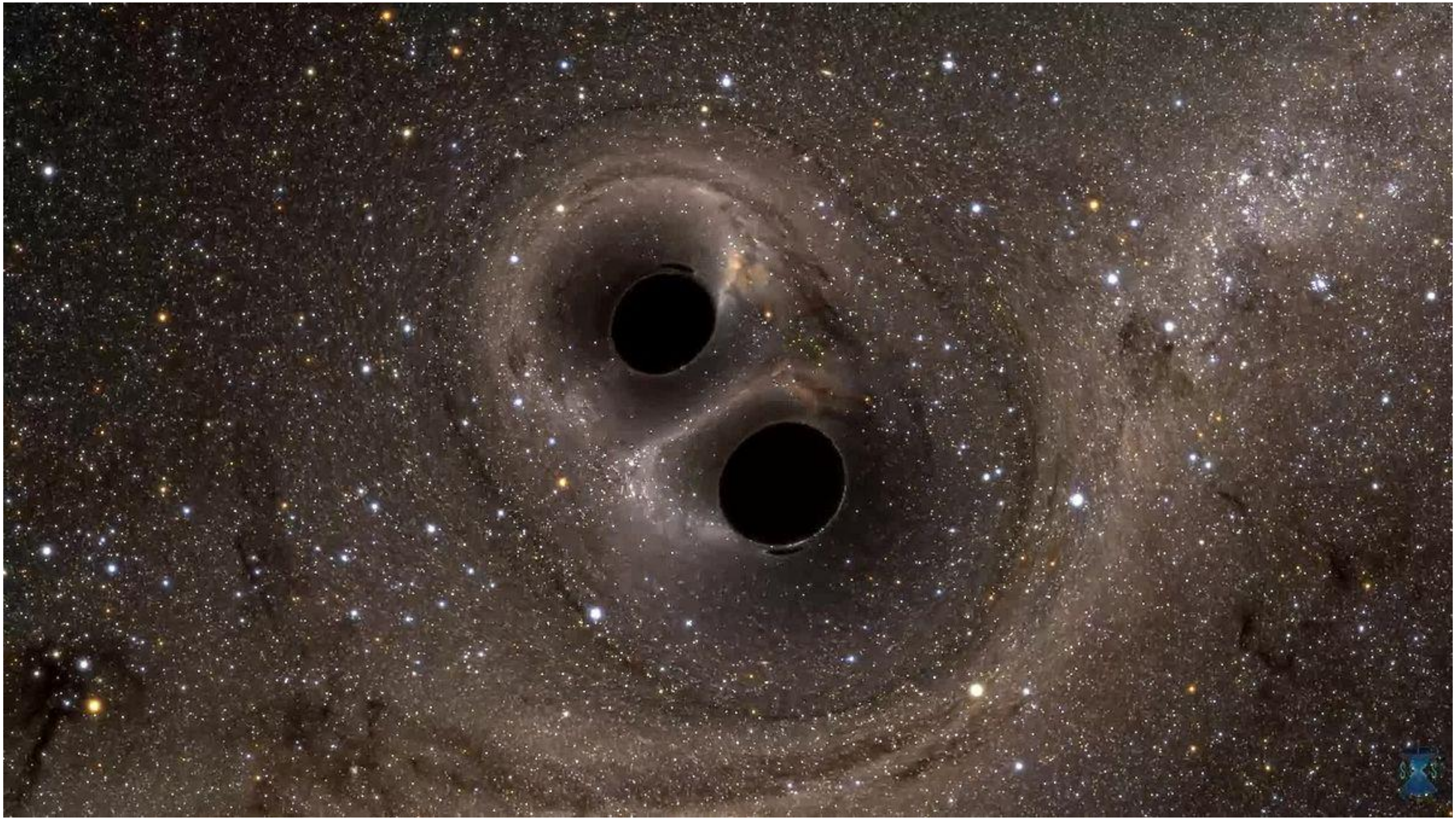
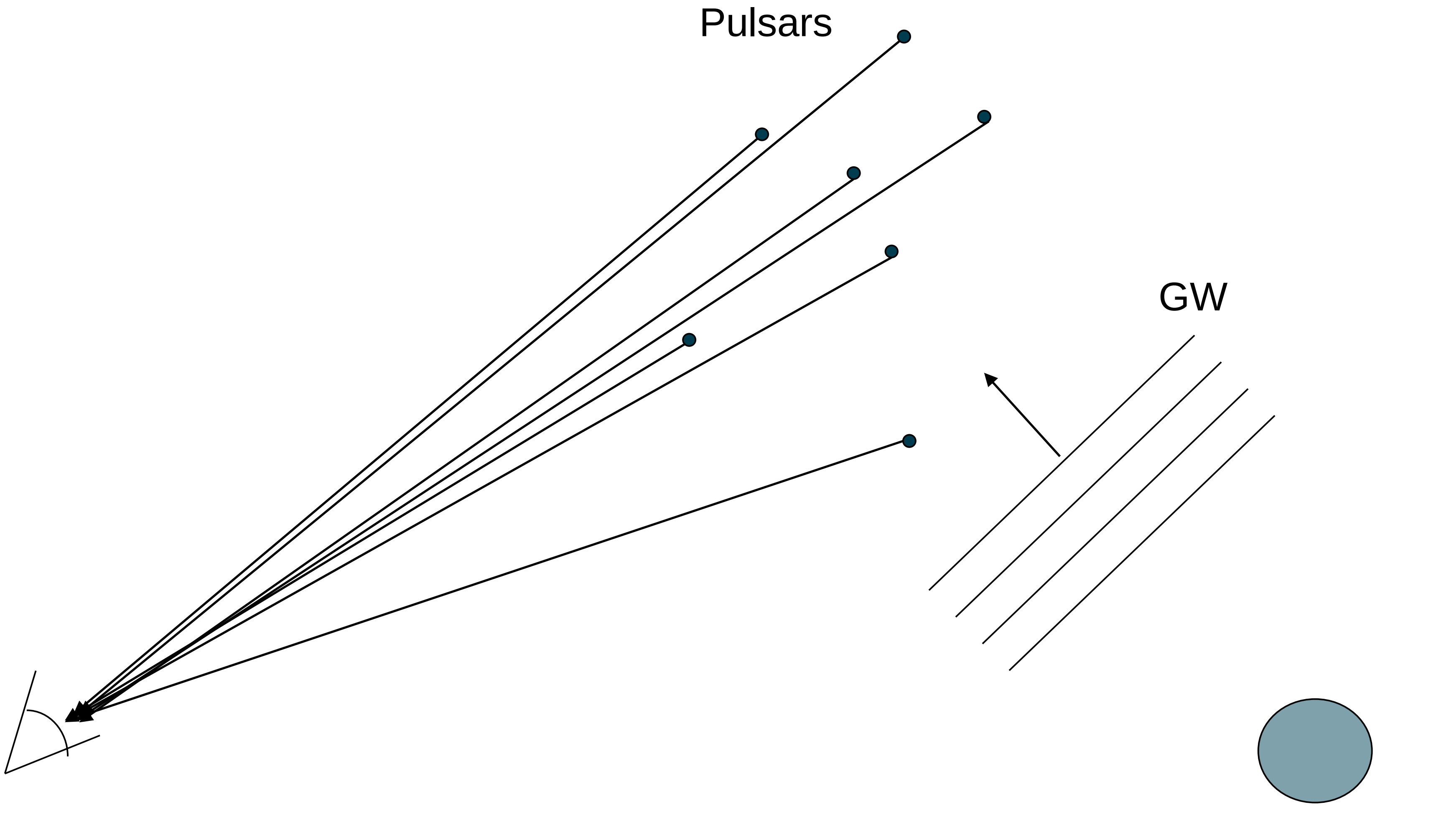
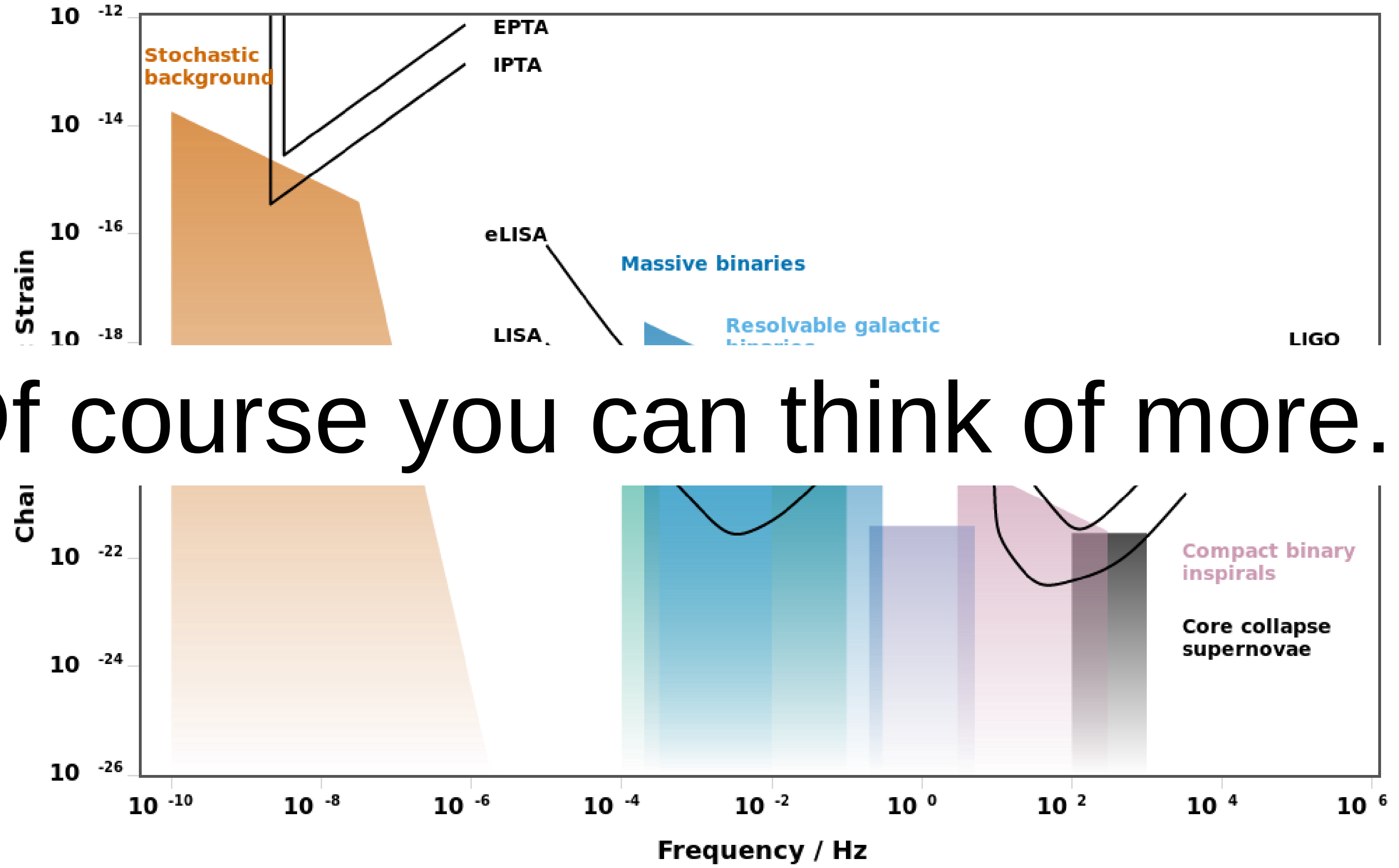
Motivation 1 – light source near a binary



Motivation 2 - Pulsar Timing Array



Motivation 2 – Pulsar Timing

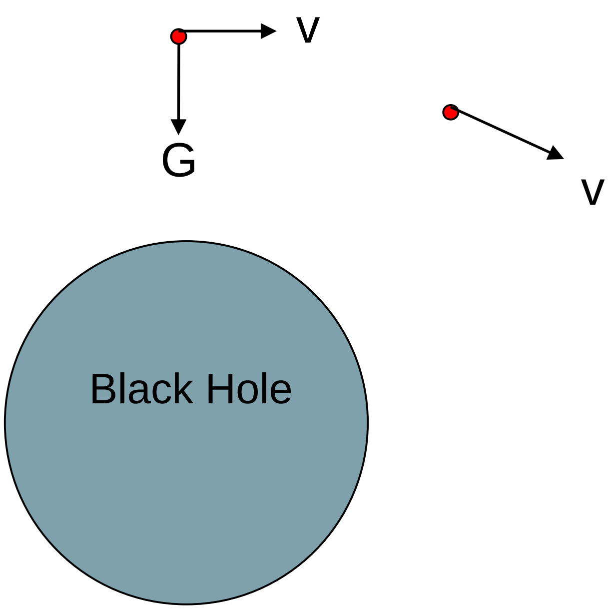


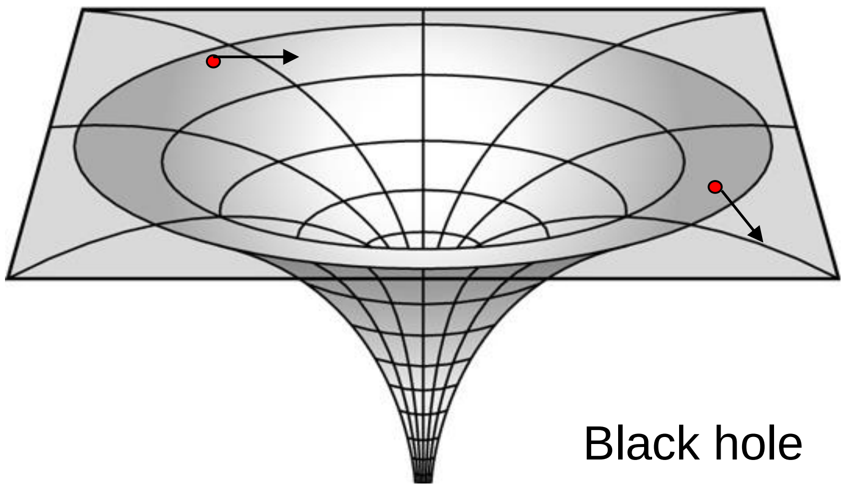
Of course you can think of more…

Moore, C., Cole, R. & Berry, C. (2013)

Motion of particles in General Relativity

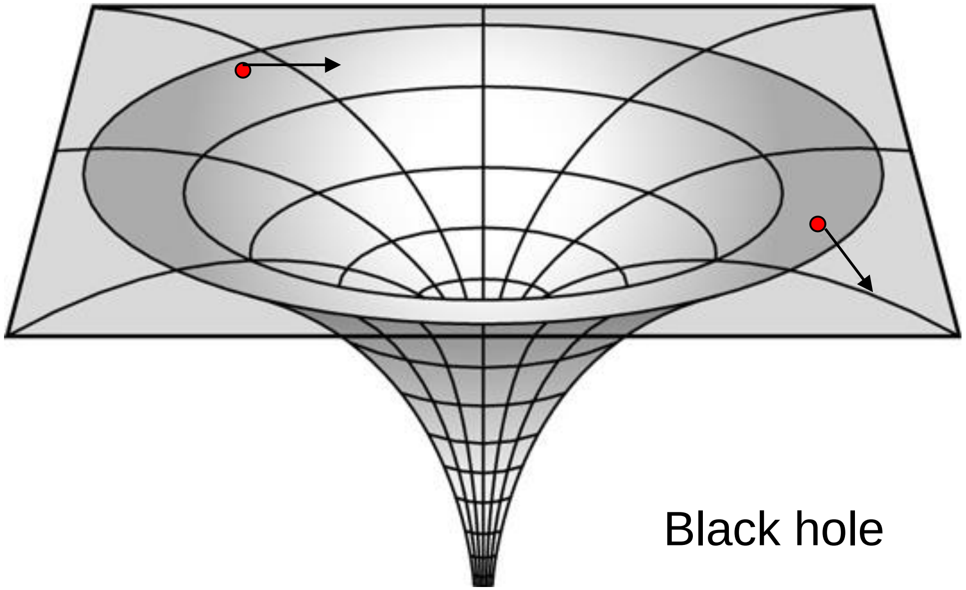
Newtonian vs. General Relativity

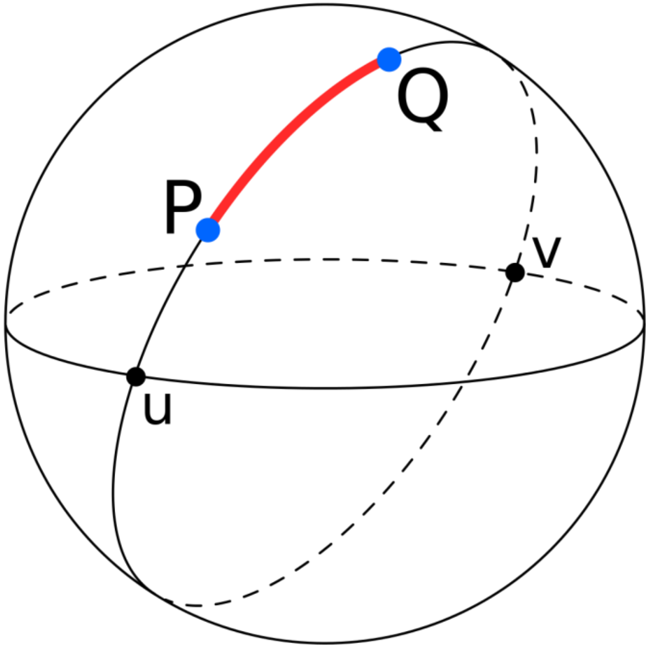




Gravitational force ⟺ Spacetime curvature

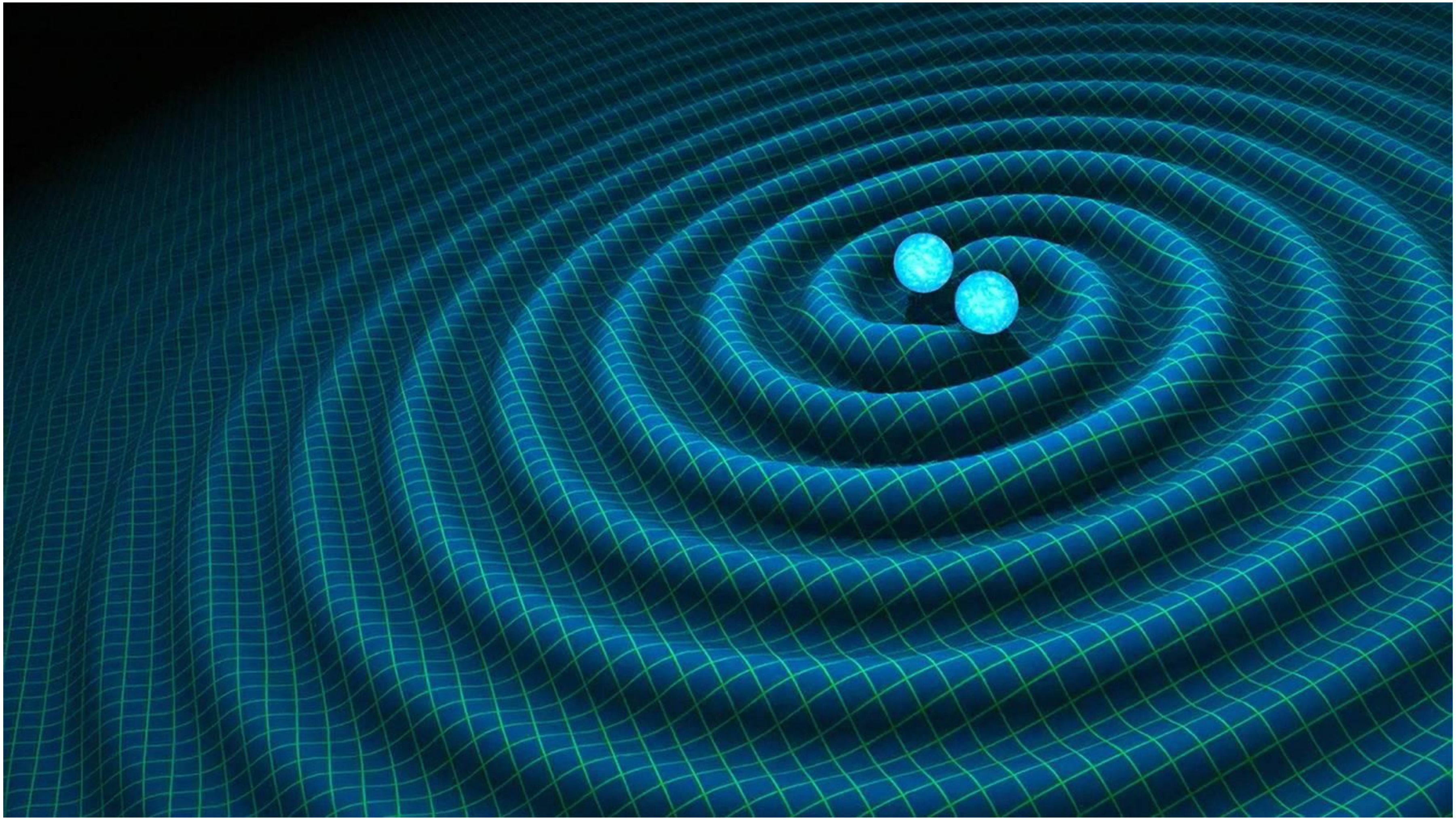
Geodesic -
a “straight” line in curved spacetime





A free-falling particle (including photon) moves along a geodesic

Gravitational waves



Gravitational waves is a propagating curvature of spacetime

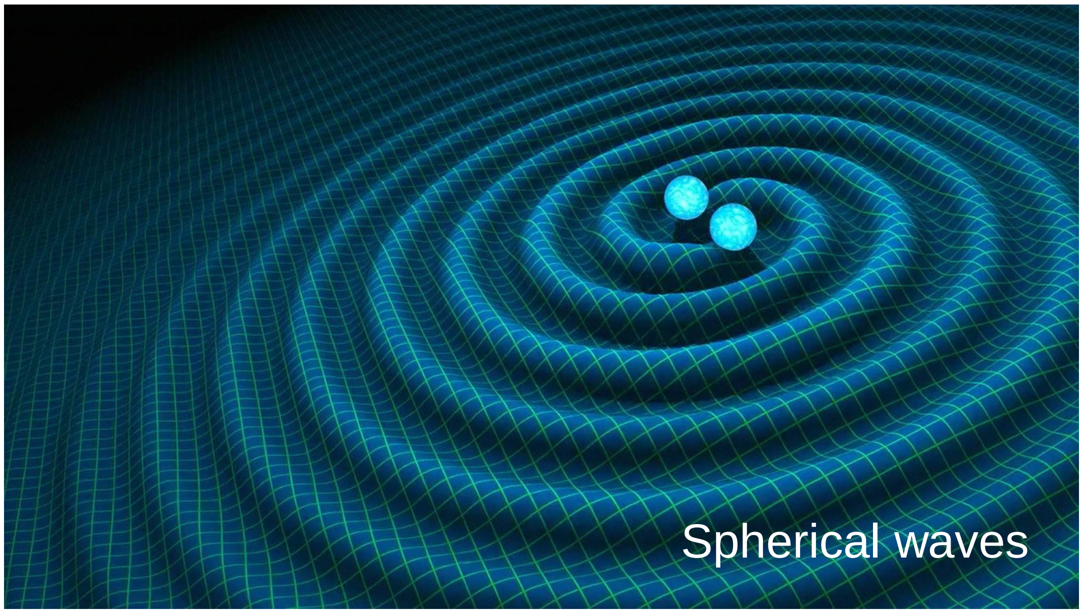
Key idea

1. Photons move in accordance with the spacetime structure

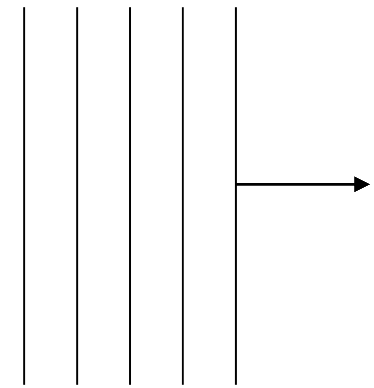
2. Gravitational waves are spacetime modulations

Calculation and preliminary results

Plane gravitational waves at far field

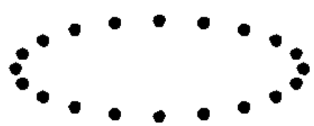


far far away…

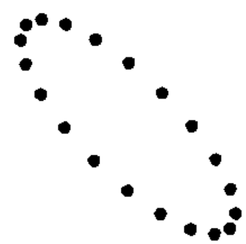


plane GW

Two polarizations



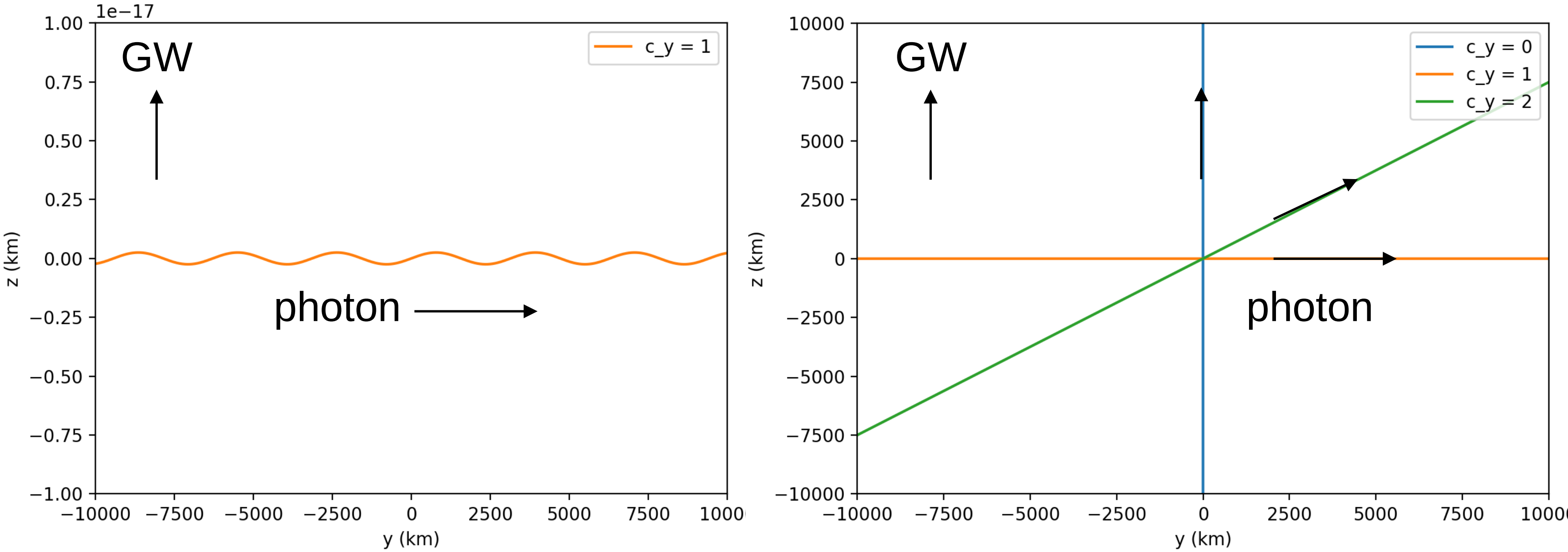
plus mode



cross mode

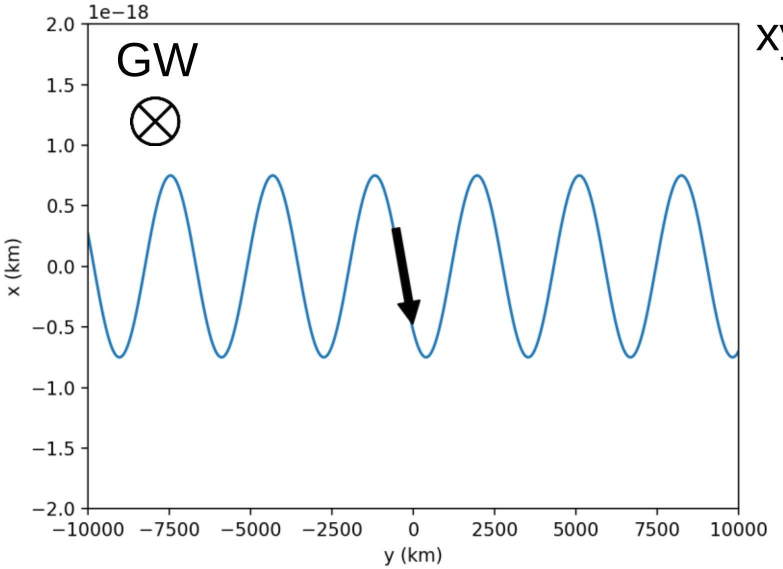
Single polarization GW - geodesic

$h_\times = 0$ $h_+ = 10^{-21}$



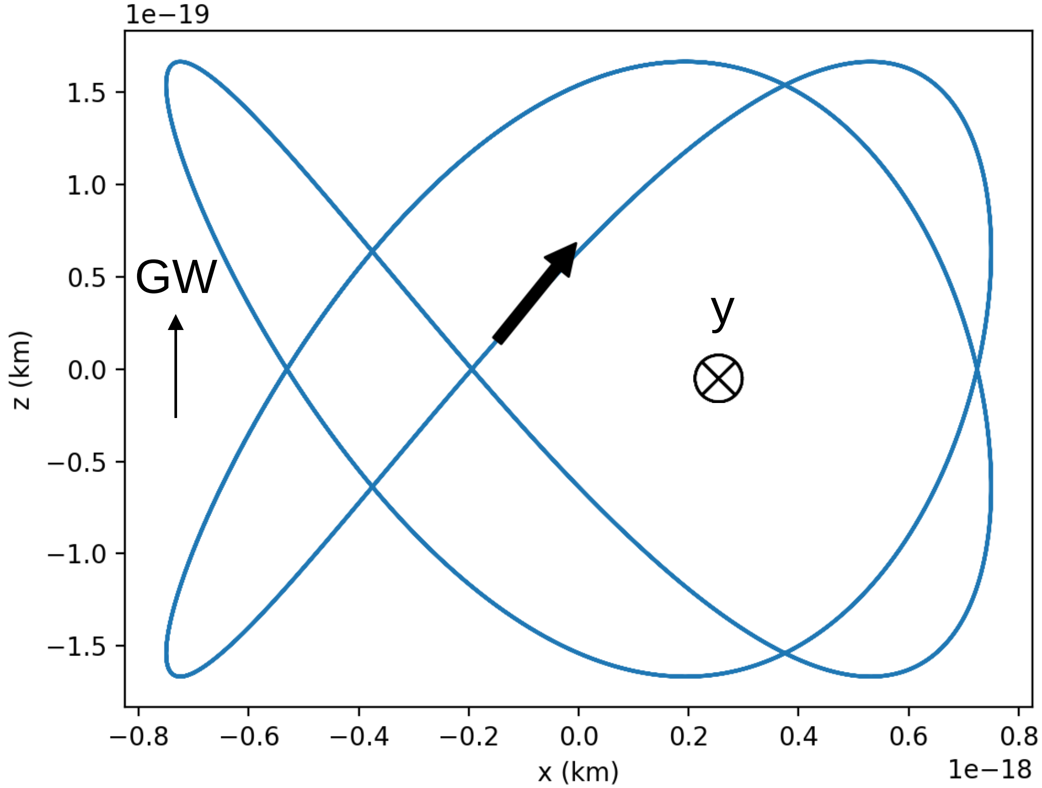
Photon propagates on yz-plane

Mixed polarization - geodesic

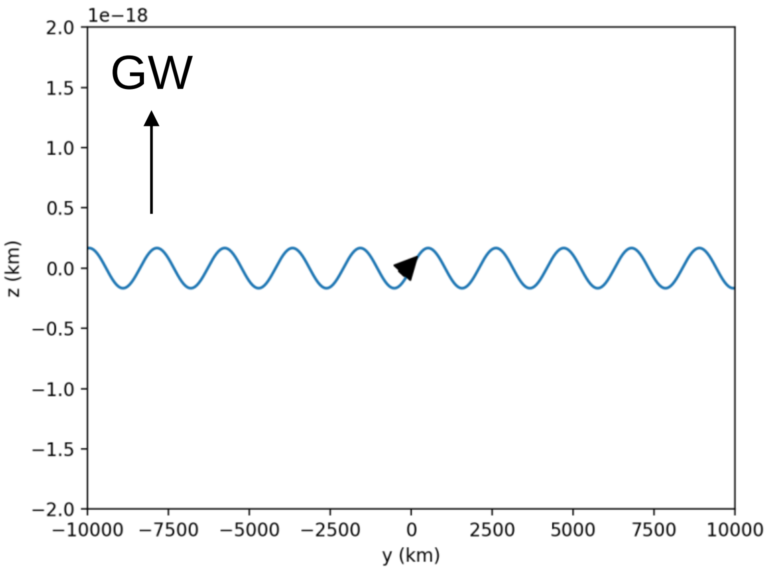


xy-plane

$$h_+ \sim h_\times \sim 10^{-21}$$



xz-plane



yz-plane

Photon propagates “in y-direction”

Remarks

- Far field approximation (on Earth),

$$h \sim 10^{-21} \Rightarrow \sim 10^{-13}\mathrm{cm}$$

- In near field,

$$h \sim 10^{-8} \Rightarrow \sim 1\mathrm{cm}$$

because

$$h \propto \frac{1}{r}$$

Future work

- Non-coupling
  - (Photon) Geodesic
  - (EM wave) Test EM field
- Coupling
  - (Photon) Spin-curvature coupling (MPD equations)
  - (EM wave) Einstein equation

Summary

- How light travels in GW is an important topic
- Geodesics of photon in plane background GW is solved
- Effect of GW on photon measurement cannot be neglected in the near field

Appendix

Plane gravitational waves at far field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$\eta_{\mu\nu}$: the flat spacetime metric

$h_{\mu\nu}$: the perturbation due to gravitational wave

$$h_+ \sim h_\times \sim 10^{-21}$$

For a z-axis propagating GW

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 + h_+ \cos k_+(z-t) & h_\times \cos(k_\times(z-t)+\phi) & 0 \\ 0 & h_\times \cos(k_\times(z-t)+\phi) & 1 - h_+ \cos k_+(z-t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Formulations

Geodesic equations: $$\ddot{x}^a + \Gamma^a_{bc}\dot{x}^b\dot{x}^c = 0$$

Null geodesic: $$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0$$

Killing vectors:

"velocity" in x or y direction

$$(1 + h_+ \cos k_+(z-t))\dot{x} + h_\times \cos[k_\times(z-t) + \phi]\dot{y} = c_x$$

$$h_\times \cos[k_\times(z-t) + \phi]\dot{x} + (1 - h_+ \cos k_+(z-t))\dot{y} = c_y$$

where c_x and c_y are conserved quantities

Exact equations of motion

$$\dot{x} = \frac{c_x - c_x h_+ \cos k_+\lambda - c_y h_\times \cos(k_\times\lambda + \phi)}{1 - h_+^2 \cos^2 k_+\lambda - h_\times^2 \cos^2(k_\times\lambda + \phi)}$$

$$\dot{y} = \frac{c_y + c_y h_+ \cos k_+\lambda - c_x h_\times \cos(k_\times\lambda + \phi)}{1 - h_+^2 \cos^2 k_+\lambda - h_\times^2 \cos^2(k_\times\lambda + \phi)}$$

$$\dot{z} = \frac{1}{2}(c_x\dot{x} + c_y\dot{y}) - \frac{1}{2}$$

$$\dot{t} = \frac{1}{2}(c_x\dot{x} + c_y\dot{y}) + \frac{1}{2}$$

$$h_+ \sim h_\times \sim 10^{-21}$$

Solutions

$$x(\lambda) = \underbrace{c_x\lambda}_{\text{0th order}} \underbrace{-\frac{c_x h_+}{k_+}\sin k_+\lambda - \frac{c_y h_\times}{k_\times}\sin(k_\times\lambda+\phi)}_{\text{1st order}}$$

$$\underbrace{+c_x(\frac{h_+^2+h_\times^2}{2}\lambda + \frac{h_+^2}{4k_+}\sin 2k_+\lambda + \frac{h_\times^2}{4k_\times}\sin(2k_\times\lambda+2\phi))}_{\text{2nd order}} \underbrace{+\mathcal{O}(\max(h_+^3, h_\times^3))}_{\text{3rd order}}$$

$$y(\lambda) = \underbrace{c_y\lambda}_{\text{0th order}} \underbrace{+\frac{c_y h_+}{k_+}\sin k_+\lambda - \frac{c_x h_\times}{k_\times}\sin(k_\times\lambda+\phi)}_{\text{1st order}}$$

$$\underbrace{+c_y(\frac{h_+^2+h_\times^2}{2}\lambda + \frac{h_+^2}{4k_+}\sin 2k_+\lambda + \frac{h_\times^2}{4k_\times}\sin(2k_\times\lambda+2\phi))}_{\text{2nd order}} \underbrace{+\mathcal{O}(\max(h_+^3, h_\times^3))}_{\text{3rd order}}$$

$$z(\lambda) = \frac{1}{2}(c_x x + c_y y) - \frac{\lambda}{2}$$

$$t(\lambda) = \frac{1}{2}(c_x x + c_y y) + \frac{\lambda}{2}$$