Investigation of Differentiable Gravitational Wave Phenomenological Model

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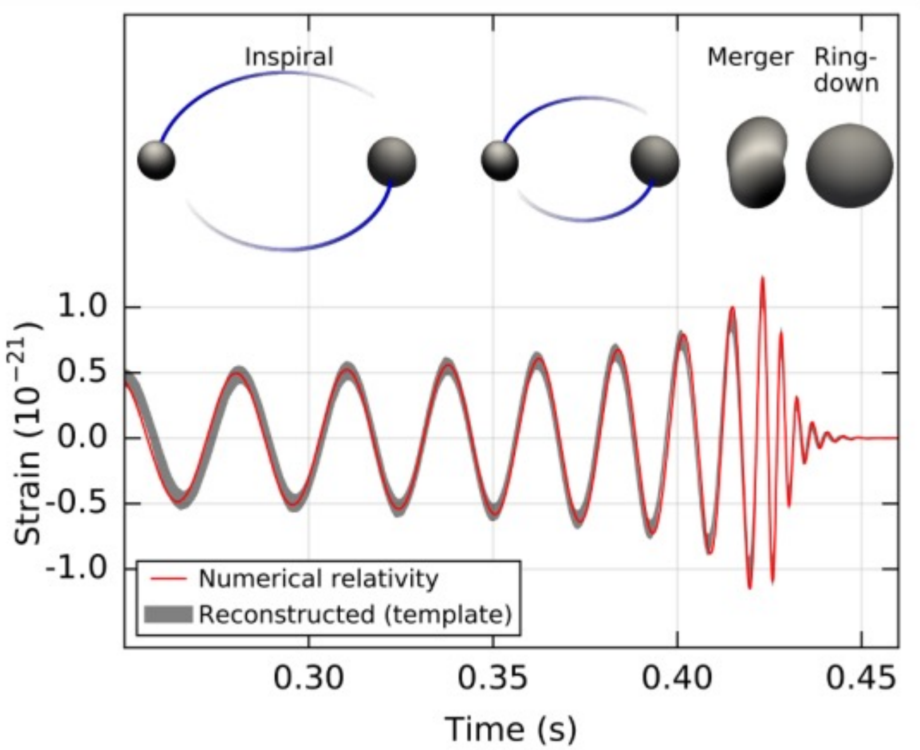
Gravitational Wave Models

- Waveform Approximations
- Used in Gravitational Wave (GW) Analyses
  - Matched Filtering, Parameter Estimation, etc.



Credit: https://doi.org/10.1103/PhysRevLett.116.061102

Why do we need these models?

- Einstein Field Equations -> Gravitational Waves
- Solve numerically?

- Computationally Expensive and Slow
  - Cannot Use Numerical Relativity (NR) waveforms for GW analysis

- Can be used to construct GW models

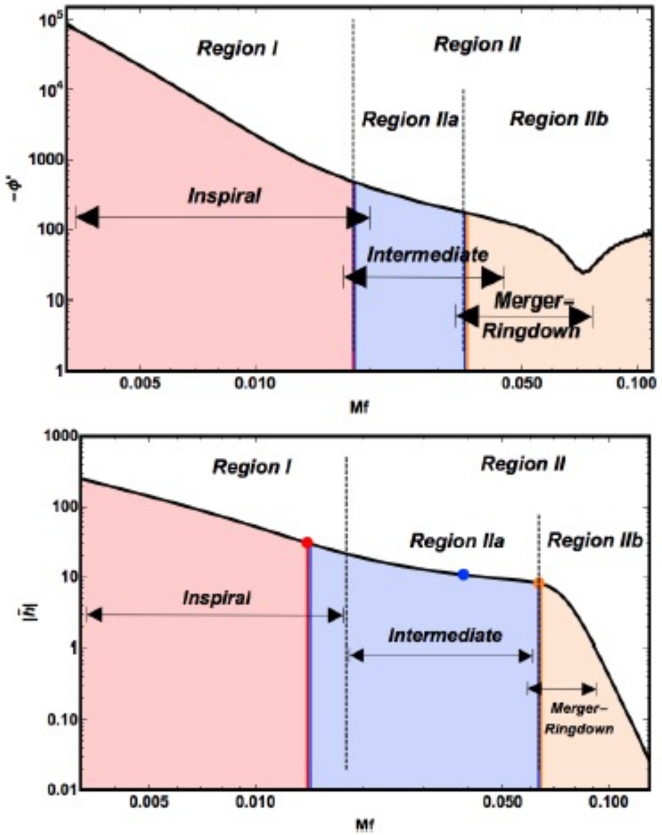
Phenomenological Model

- Not derived purely from theory
- Constructed using specific ansatz with tunable parameters that is fitted to NR waveforms

IMRPhenomD

- Non-precessing, similar spin
- Parameterized by black hole mass ratio $q = m_1/m_2$ and effective spin χ_{eff}

- Constructed in segments
  - Inspiral, Intermediate, Merger-Ringdown
  - Fitted in segments

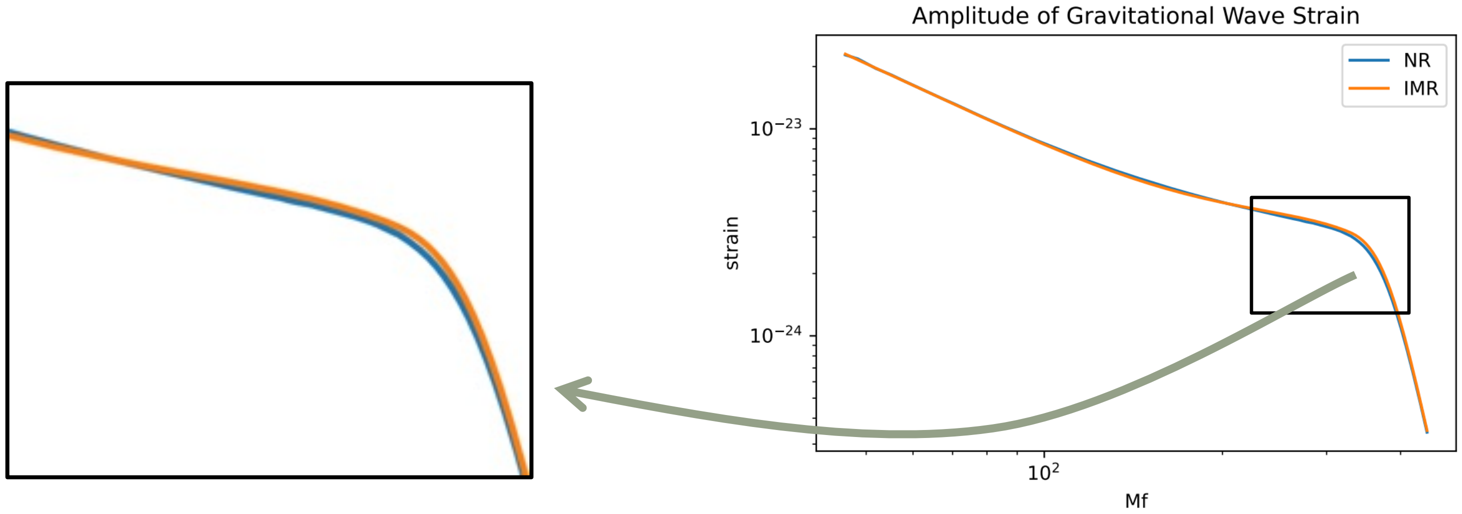


Sketch of the ansatz of IMRPhenomD model

Objectives

- Improve the current model by re-fitting tunable parameters
  - Comparing with ~10 testing waveforms

- Analyze the model to determine its limitations



Method

- The mismatch between two waveforms is defined as

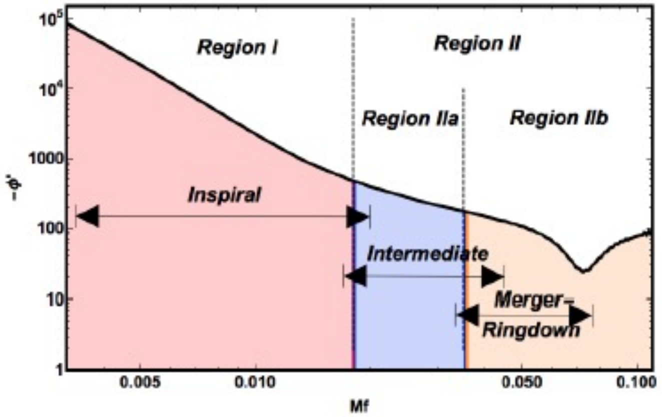
$$\mathcal{M}(h_1, h_2) = 1 - \max_{t_0,\phi_0}\left[\mathrm{Re}\int_{f_{\min}}^{f_{\max}} \hat{h}_1(\vec{\lambda}, f)\hat{h}_2^*(\vec{\lambda}, f)e^{i(2\pi f t_0+\phi_0)}\, df\right]$$

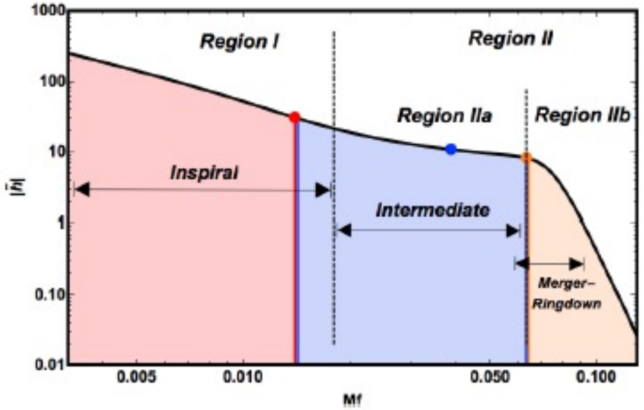
- Quantify the error between two waveforms h_1 and h_2
  - IMRPhenomD vs NR waveforms
  - Used widely in GW analysis

Method

Mean Square Error

- Previous loss function ≠ mismatch
  - Tunable parameters not at minimum mismatch
- Gradient Descent $\boldsymbol{\lambda}_{k+1} = \boldsymbol{\lambda}_k - \alpha \nabla \mathcal{M}(\boldsymbol{\lambda})$
  - $\boldsymbol{\lambda}$ is a 209-dimensional vector which contains all the tunable parameters



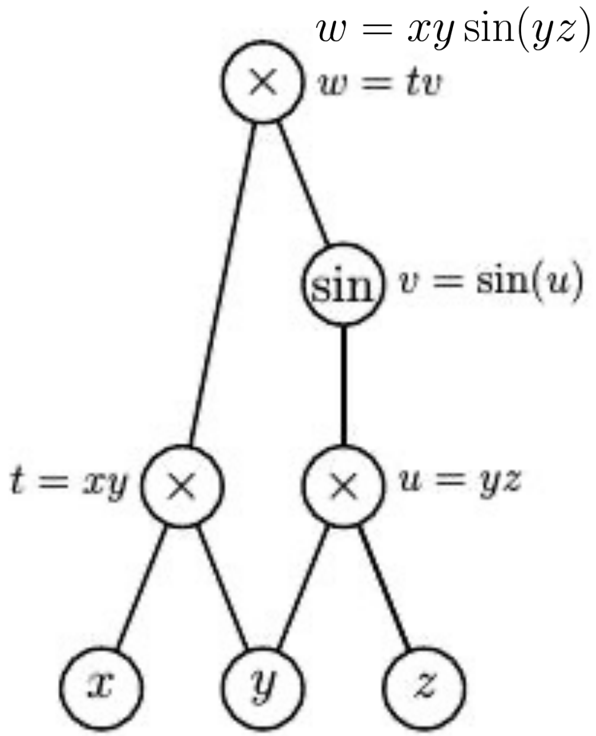


Sketch of the ansatz of IMRPhenomD model

Automatic Differentiation

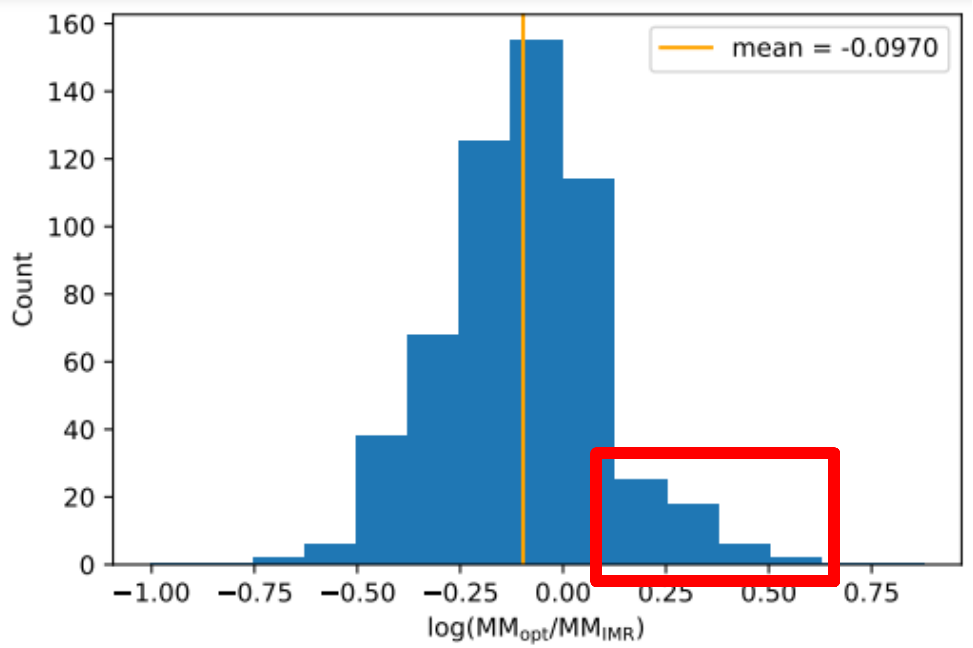
- Chain Rule
- Differentiate simple functions, then compose them back

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial w}{\partial u}\frac{\partial u}{\partial x}$$



Preliminary Result

- $\sim 20\%$ decrease in mismatch
- Some waveforms have worse performance after optimization



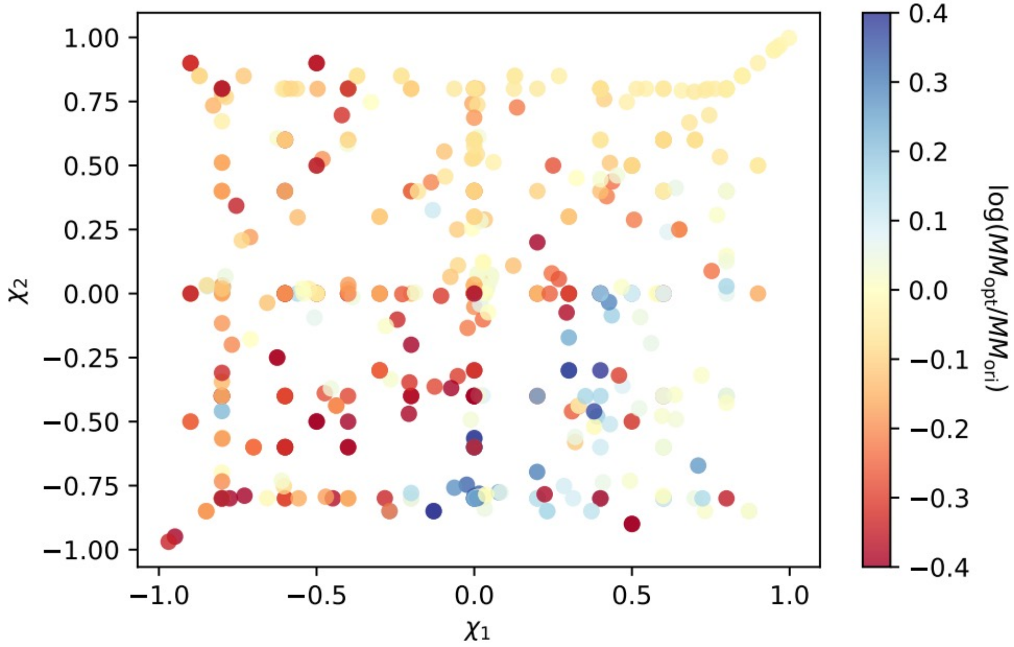
Distribution of the change in mismatch before and after optimization

Preliminary Result

- Model is parameterized by effective spin χ_{eff}

$$\chi_{\text{eff}} = \frac{m_1\chi_1 + m_2\chi_2}{m_1 + m_2}$$

- Degeneracy in χ_1 and χ_2



Scatter plot of testing waveforms in parameter space. Color represents the change in mismatch before and after optimization

Further Investigation

- Perform similar analysis to newer waveform models (IMRPhenomXAS)
- Analyze the effect of precession

Q&A