# EXTENDING SUPERCONFORMAL VECTOR FIELDS ON 6D SUPER-MINKOWSKI SPACETIME TO 10D

**Speaker: Siew Rui Xian**

**Supervisor: Professor Jim Gates**

# SURE 2019

- Summer Student Theoretical Physics Research Session (SSTPRS) at Brown University
- Supervisor: Professor Jim Gates

# MOTIVATION

- Why do we study conformal field theories (CFT)?

-AdS/CFT correspondence and superstring theory

# INTRODUCTION

# BASIC ALGEBRA

- What is a group?
- What is a vector space?
- What is a Lie group?
- What is a Lie algebra?
- What is a representation?

# GROUPS

A group $(G, \circ)$ is a set $G$ equipped with a binary operation $\circ : G \to G$ that satisfy three properties:

1. Associativity: $f \circ (g \circ h) = (f \circ g) \circ h$
2. Identity: $\exists e \in G$ such that $g \circ e = e \circ g = g \quad \forall g \in G$
3. Inverse: $\forall g \in G, \exists g^{-1}$ such that $g \circ g^{-1} = g^{-1} \circ g = e$

E.g. $(\mathbb{R}, +)$ and $(U(1), \cdot)$

- A Lie group is a group that is also a differentiable manifold, such that its group operation and inverse operation are smooth.

# VECTOR SPACE

A real vector space is a set $V$ equipped with a commutative binary operation (usually called addition) $+ : V \times V \to V$ that makes $(V, +)$ into an abelian group, and a scalar multiplication $\cdot : \mathbb{R} \times V \to V$ that satisfies the following properties $\forall a, b \in \mathbb{R}$ and $\forall v, u \in V$:

1. $(a + b) \cdot v = a \cdot v + b \cdot v$
2. $a \cdot (v + u) = a \cdot v + a \cdot u$
3. $a \cdot (b \cdot v) = (ab) \cdot v$
4. $1 \cdot v = v$

E.g. $(\mathbb{R}^n, +, \cdot)$ and $(\mathbb{C}^n, +, \cdot)$

# REPRESENTATIONS

- Homomorphisms are structure preserving maps.

$$\rho : G \to G'$$
$$\rho(gh) = \rho(g)\rho(h)$$
$$\rho(e) = e'$$

- Let A be some algebraic structure (group, algebra, etc). Then a representation of A is a homomorphism ρ: A → Gl(V), where V is some vector space. Gl(V) is the group of isomorphism from V to itself.

# LIE ALGEBRA (*ABSTRACT*)

A real Lie algebra is a real vector space $V$ with a binary operation (called a Lie bracket) $[\cdot,\cdot] : V \times V \to V$ that satisfy three properties $\forall u, v, w \in V$ and $\forall r \in \mathbb{R}$:

1. $[u, v] = -[v, u]$
2. $[ru + v, w] = r[u, w] + [v, w]$
3. $[u, [v, w]] = [[u, v], w] + [v, [u, w]]$

The third condition is called the Jacobi identity. It means that the Lie bracket makes the Lie algebra into a representation of itself.

# EXAMPLES OF LIE ALGEBRA

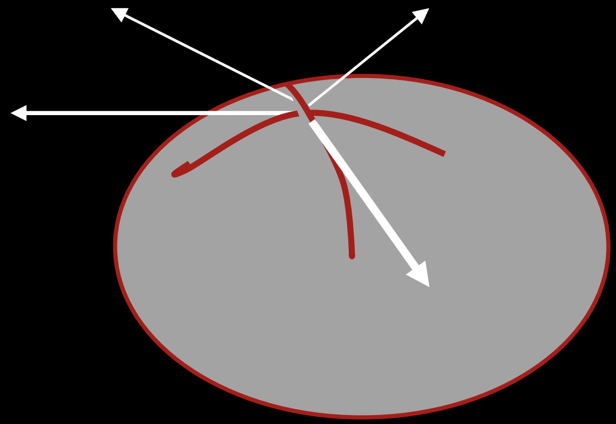
1. The commutator $[a, b] := ab - ba$ together with $M_{n\times n}(\mathbb{R})$ or any matrix ring

2. The Poisson bracket in classical mechanics with the ring of all smooth functions $C^\infty(\mathbb{R}^{2n})$ on a phase space $\mathbb{R}^{2n}$

3. The commutator and the tangent vectors at the identity element of any Lie groups

# LIE ALGEBRA OF A LIE GROUP

- Tangent vectors X at the identity element $e$ of a Lie group $G$
- Related to the Lie group by the exponential map.



$$e^{tX} = \varphi(t),$$

Where $\varphi\colon I \to G$ is the unique curve satisfying
$\varphi\,(0) = e$ and $\varphi'(0) = X$

# LIE SUPERALGEBRA

A real Lie superalgebra is a $\mathbb{Z}_2$-graded real associative algebra $A = A_0 \oplus A_1$ with a graded derivation (called the superbracket) $[\cdot,\cdot\} : A \times A \to A$ satisfying the properties:

1. If $a_i \in A_i$ and $a_j \in A_j$ for some $i, j \in \{0, 1\}$, then $a_i a_j \in A_{(i+j)mod2}$
2. If $a_0 \in A_0$, then $a_0 b = b a_0 \; \forall b \in A$
3. If $a_1, b_1 \in A_1$, then $a_1 b_1 = -b_1 a_1$
4. $[a, b\} = -(-1)^{|a|\cdot|b|}[b, a\}$, where $|x| = 0$ if $x \in A_0$ and $|x| = 1$ if $x \in A_1$
5. $(-1)^{|a|\cdot|c|}[a, [b, c\}\} + (-1)^{|b|\cdot|a|}[b, [c, a\}\} + (-1)^{|b|\cdot|c|}[c, [a, b\}\} = 0$

- Fact: The even part of a Lie superalgebra forms an ordinary Lie algebra.

# CLIFFORD ALGEBRA

A standard real Clifford algebra $\mathscr{C}_{t,s}$ is a unital associative real algebra freely generated by the set $V = \{e_0, e_1, ..., e_{t+s-1}\}$ such that

$$e_i e_j + e_j e_i = 2\eta_{ij}1 \quad \forall i, j \in \{1, ..., n\}, \text{where } \eta_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ -1 \text{ if } 0 \leq i = j \leq t-1 \\ 1 \text{ if } t \leq i = j \leq t+s-1 \end{cases}$$

Hence, $\mathscr{C}_{t,s} = span\{1, e_0, ..., e_{s+t-1}, e_0 e_1, ..., e_0 ... e_{s+t-1}\}$

# CLIFFORD ALGEBRA

- Why do we care? Because it is a representation of the **Lorentz group**
- It is a superalgebra
- Example: Pauli matrices (a basis for su(2))

$$\tilde{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tilde{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tilde{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tilde{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In higher dimension, we have sigma matrices (will be shown later) that are built from Pauli matrices.

# CLASSIFICATION OF CLIFFORD ALGEBRA

| $t - s \bmod. 8$ | $\mathcal{C}_{t,s}$ |
|---|---|
| 0 | $\mathcal{M}_{2^\ell}(\mathbb{R})$ |
| 1 | $\mathcal{M}_{2^\ell}(\mathbb{R}) \oplus \mathcal{M}_{2^\ell}(\mathbb{R})$ |
| 2 | $\mathcal{M}_{2^\ell}(\mathbb{R})$ |
| 3 | $\mathcal{M}_{2^\ell}(\mathbb{C})$ |
| 4 | $\mathcal{M}_{2^{\ell-1}}(\mathbb{H})$ |
| 5 | $\mathcal{M}_{2^{\ell-1}}(\mathbb{H}) \oplus \mathcal{M}_{2^{\ell-1}}(\mathbb{H})$ |
| 6 | $\mathcal{M}_{2^{\ell-1}}(\mathbb{H})$ |
| 7 | $\mathcal{M}_{2^\ell}(\mathbb{C})$ |

Taken from Rausch de Traubenberg's "Clifford Algebra in Physics"

# POINCARÉ GROUP

- Lorentz group: Homogeneous part of the Poincaré group.

$$x' = Ax + c$$

- Poincaré group: Isometry group of Minkowski spacetime, i.e. infinitesimal automorphisms (symmetries) that keep the metric tensor invariant.

$$\mathcal{L}_X g = 0 \iff \partial_\mu X_\nu + \partial_\nu X_\mu = 0$$

- In our 4D spacetime, its identity component is 5(4)/2=10 dimensional (as a manifold).

$$ISO(1, d-1) = \mathbb{R}^d \rtimes O(1, d-1)$$

$$\mathfrak{iso}(\mathbb{R}^{1,d-1}) = \mathbb{R}^d \oplus \mathfrak{so}(1, d-1)$$

# CONFORMAL GROUP

- Angle-preserving transformations, an extension of the Poincaré group

$$x^\mu \to x^\mu + \xi^\mu$$

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{d}\eta_{\mu\nu}\partial \cdot \xi$$ where $\partial \cdot \xi = \partial^\rho \xi_\rho$.

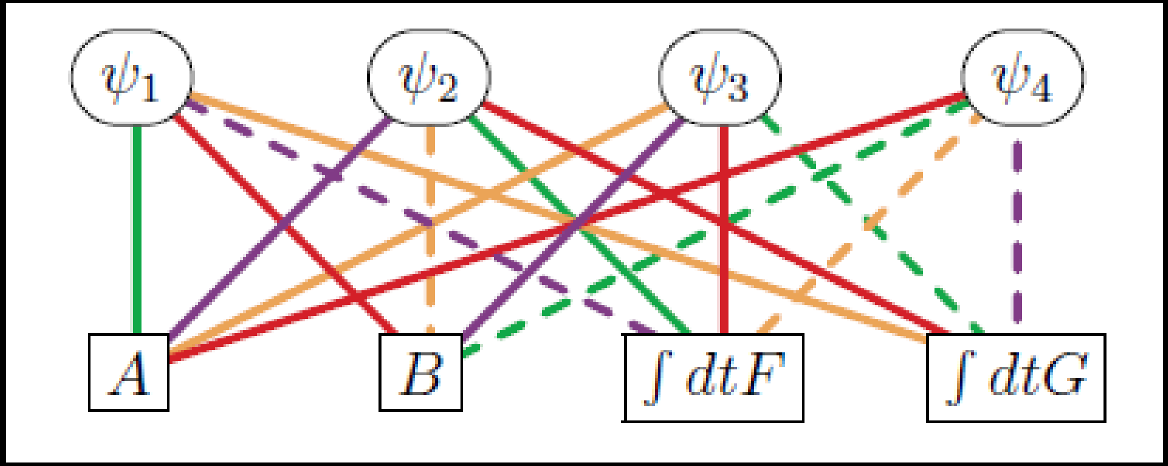
- Conformal algebra (group )isomorphic to SO algebra (group)

- Conf(1,d-1) ~SO(2,d)~AdS(d+1), the d+1 Lorentzian Anti-de-Sitter space embedded as a surface in the d+2 D spacetime.

$$\mathfrak{conf}(1, d-1) \simeq \mathfrak{so}(2, d)$$

# SUPERSYMMETRY

- Coleman-Mandula Theorem: Symmetry group of a 4D QFT can only have Poincaré group and internal symmetries.
- Haag-Lopuszanski-Sohnius Theorem: We can have anticommuting symmetries too!
- Transformation between bosons and fermions




Adinkra, taken from Gates et al, "Adinkra Height Yielding Matrix Numbers: Eigenvalue Equivalence Classes for Minimal Four-Color Adinkras"

# SUPERSPACE (SUPERMANIFOLDS)

The ordinary Minkowski spcetime can be recovered by

$$\mathbb{R}^{1,d-1} \simeq \mathfrak{iso}(\mathbb{R}^{1,d-1})/\mathfrak{so}(1,d-1)$$

We can imagine a Minkowski supespace defined as above but replace the Poincaré algebra by Super-Poincaré algebra, which is a superalgebra that contains the Poincaré algebra as its even part.

The topology of supermanifolds is weird, e.g. not Hausdorff, and integration=differentiation

# Grassmann Numbers

- Objects that anticommute:

$$\theta_i\theta_j = -\theta_j\theta_i$$

- Mathematically, the Grassmann algebra generated by a complex vector space V is

$$\Lambda(V) := \mathbb{C} \oplus \Lambda^1 V \oplus \Lambda^2 V \oplus \cdots \oplus \Lambda^n V$$

where $\Lambda^2 V = V \wedge V$ and $n := dim_{\mathbb{C}}(V)$

- On a super-Minkowski space, we have ordinary (bosonic) spacetime coordinates $x^a$ and anticommuting (fermionic) coordinates $\theta^\alpha$

# SUPERCONFORMAL VECTOR FIELDS

# WHAT DO WE WANT?

1. A representation of some superalgebra that acts as infinitesimal automorphism on super-Minkowski space, i.e. vector field representations, that is
2. An extension of the super- Poincaré algebra, and
3. When its even (bosonic) part's action is restricted on ordinary Minkowski space, it reduces to the ordinary conformal algebra so(2,d)

# SUPER-POINCARÉ ALGEBRA

$$P_a = \partial_a$$

$$M_{ab} = -\frac{1}{2}x_a\partial_b + \frac{1}{2}x_b\partial_a - \frac{1}{4}\theta^\beta(\sigma_{ab})_\beta{}^\alpha\partial_\alpha$$

$$Q_\alpha = \partial_\alpha - \frac{1}{2}\theta^\beta(\sigma^a)_{\alpha\beta}\partial_a$$

$$[P_a, M_{bc}] = -\eta_{a[b}P_{c]}$$

$$[M_{ab}, M_{cd}] = \eta_{c[a}M_{b]d} - \eta_{d[a}M_{b]c}$$

$$[M_{ab}, Q_\alpha] = \frac{1}{4}(\sigma_{ab})_\alpha{}^\beta Q_\beta$$

Generated by translation, Lorentz rotation, and supercharge Q

# 5+1D CONFORMAL VECTOR FIELDS

$$D = x \cdot \partial + \frac{1}{2}\theta \cdot \partial$$

$$P_a = \partial_a$$

$$M_{ab} = -\frac{1}{2}x_a\partial_b + \frac{1}{2}x_b\partial_a - \frac{1}{4}\theta_c^\beta(\sigma_{ab})_\beta{}^\alpha\partial_\alpha^c$$

$$K_a = x^2\partial_a - 2x_a x \cdot \partial + \frac{1}{4}(\sigma_a)_{\beta\alpha}\theta^\alpha \cdot \theta^\gamma\theta^\beta \cdot \theta^\delta(\sigma_b)_{\gamma\delta}\partial_b - x_a\theta \cdot \partial$$

$$- x^b\theta_c^\alpha(\sigma_{ab})_\alpha{}^\gamma\partial_\gamma^c + i(\sigma_a)_{\beta\alpha}\theta_c^\alpha\theta^\beta \cdot \theta^\gamma\partial_\gamma^c$$

Dilation, translation, Lorentz transformation, special conformal transformation

# CONFORMAL ALGEBRA

$$[P_a, M_{bc}] = -\eta_{a[b}P_{c]}$$

$$[K_a, D] = -K_a$$

$$[P_a, D] = P_a$$

$$[K_a, M_{bc}] = -\eta_{a[b}K_{c]}$$

$$[M_{ab}, M_{cd}] = \eta_{c[a}M_{b]d} - \eta_{d[a}M_{b]c}$$

$$[P_a, K_b] = -4M_{ab} - 2\eta_{ab}D$$

This is true for all dimension >2 ! The indices may come from any range.

# FERMIONIC VECTOR FIELDS

Q supercharge: the "square-root" of the translation P

$$Q_\alpha^a = \partial_\alpha^a - \frac{1}{2} i \Omega^{ac} \theta_c^\gamma (\sigma^b)_{\alpha\gamma} \partial_b$$

S supercharge: the "square-root" of the special conformal transformation K

$$S_a^\alpha = -\Omega_{ac}((\sigma^b)^{\alpha\gamma} x_b + i\theta^\alpha \cdot \theta^\gamma)\partial_\gamma^c + 2i\theta_a^\gamma \theta_c^\alpha \partial_\gamma^c + \frac{1}{2} i x_b \theta_a^\gamma (\sigma^{db})_\gamma{}^\alpha \partial_d$$
$$+ \frac{1}{2} i \theta_a^\alpha x \cdot \partial - \frac{1}{2} (\sigma^d)_{\gamma\delta} \theta_a^\gamma \theta^\delta \cdot \theta^\alpha \partial_d$$

$$\{S_a^\alpha, S_b^\beta\} = -2i\Omega_{ab}K^{\alpha\beta}$$
$$\{Q_\alpha^a, Q_\beta^b\} = -2i\Omega^{ab}P_{\alpha\beta}$$

where the R-symmetry invariant tensor satisfies $\Omega_{ab}\Omega^{bc} = \delta_a{}^c$.

$$\{Q_\alpha^a, S_b^\beta\} = i\delta_\alpha{}^\beta(\delta^a{}_b D - 4\Omega_{bc}U^{ac}) + 2i\delta^a{}_b M_\alpha{}^\beta$$

R- symmetry bosonic vector fields: $U^{ab}$

# 9+1D SPACETIME: SIGMA MATRICES

$$(\sigma^0)_{\alpha\beta} = \tilde{\sigma}^0 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^0,$$
$$(\sigma^1)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^2 \otimes \tilde{\sigma}^2 \otimes \tilde{\sigma}^2,$$
$$(\sigma^2)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^2 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^1,$$
$$(\sigma^3)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^2 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^1,$$
$$(\sigma^4)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^1 \otimes \tilde{\sigma}^2 \otimes \tilde{\sigma}^0,$$
$$(\sigma^5)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^3 \otimes \tilde{\sigma}^2 \otimes \tilde{\sigma}^0,$$
$$(\sigma^6)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^1 \otimes \tilde{\sigma}^2,$$
$$(\sigma^7)_{\alpha\beta} = \tilde{\sigma}^2 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^3 \otimes \tilde{\sigma}^2,$$
$$(\sigma^8)_{\alpha\beta} = \tilde{\sigma}^1 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^0,$$
$$(\sigma^9)_{\alpha\beta} = \tilde{\sigma}^3 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^0 \otimes \tilde{\sigma}^0$$

$$(\sigma^{ab})_\alpha{}^\gamma := (\sigma^{[a}\sigma^{b]})_\alpha{}^\gamma,$$
$$(\sigma^{abc})_{\alpha\gamma} := (\sigma^{bc})_\alpha{}^\beta(\sigma^a)_{\beta\gamma} + 2\eta^{a[b}(\sigma^{c]})_{\alpha\gamma},$$
$$(\sigma^{abcd})_\alpha{}^\gamma := (\sigma^a)_{\alpha\beta}(\sigma^{bcd})^{\beta\gamma} - 3\eta^{a[b}(\sigma^{cd]})_\alpha{}^\gamma,$$
$$(\sigma^{abcde})_{\alpha\gamma} := (\sigma^{bcde})_\alpha{}^\beta(\sigma^a)_{\beta\gamma} + 4\eta^{a[b}(\sigma^{cde]})_{\alpha\gamma}$$

$$(\sigma^a)_{\alpha\beta}(\sigma^b)^{\beta\gamma} + (\sigma^b)_{\alpha\beta}(\sigma^a)^{\beta\gamma} = 2\eta^{ab}\delta_\alpha{}^\gamma$$

In 10D we have Majorana-Weyl spinors, hence the sigma matrices are 16 times 16 real matrices.

# FIERZ IDENTITIES

$$(\sigma^b)_{\alpha[\beta}(\sigma_{ab})_{\gamma]}{}^{\delta} = \frac{5}{32}(\sigma^{[2]})_{\alpha}{}^{\delta}(\sigma_{a[2]})_{\beta\gamma} - \frac{1}{32}(\sigma_{a[3]})_{\alpha}{}^{\delta}(\sigma^{[3]})_{\beta\gamma}$$

$$(\sigma^{[3]}\sigma^a)_{\gamma}{}^{(\alpha}(\sigma_{[3]})^{\beta)\delta} = -3(\sigma_b)^{\alpha\beta}(\sigma^{ab})_{\gamma}{}^{\delta} + \frac{1}{8}(\sigma^{a[4]})^{\alpha\beta}(\sigma_{[4]})_{\gamma}{}^{\delta} - 45(\sigma^a)^{\alpha\beta}\delta_{\gamma}{}^{\delta}$$

$$(\sigma^{a[3]})_{[\delta}{}^{\alpha}(\sigma_{[3]})_{\beta]\gamma} = -\frac{1}{2}(\sigma_{[3]})_{\delta\beta}(\sigma^{a[3]})_{\gamma}{}^{\alpha}$$

$$(\sigma^{a[3]})_{\delta}{}^{[\alpha}(\sigma_{[3]})^{\beta]\gamma} = -\frac{1}{2}(\sigma_{[3]})^{\alpha\beta}(\sigma^{a[3]})_{\delta}{}^{\gamma}$$

$$(\sigma_{cde}\sigma_{ab})^{[\beta\alpha]} = 2\eta_{c[b}(\sigma_{a]de})^{\alpha\beta} - 2\eta_{d[b}(\sigma_{a]ce})^{\alpha\beta} + 2\eta_{e[b}(\sigma_{a]cd})^{\alpha\beta}$$

# RESULTS

**Proposition 1.** *The generalized expression of $S^\alpha$ satisfying*

$$[S^\alpha, D] = -\frac{1}{2}S^\alpha, \tag{5.14}$$

$$[P_a, S^\alpha] = (\sigma^a)^{\alpha\beta}Q_\beta \tag{5.15}$$

*is given by*

$$S^\alpha = x^a(\sigma_a)^{\alpha\beta}Q_\beta + k_1\theta^\alpha\theta\cdot\partial + k_2(\sigma^{abc})_{\beta\gamma}(\sigma_{abc})^{\alpha\delta}\theta^\beta\theta^\gamma\partial_\delta + k_3(\sigma^{abc})_{\beta\gamma}(\sigma_{bc})_\delta{}^\alpha\theta^\delta\theta^\beta\theta^\gamma\partial_a + k_4(\sigma_{bcd})_{\beta\gamma}(\sigma^{abcd})_\delta{}^\alpha\theta^\delta\theta^\beta\theta^\gamma\partial_a \tag{5.16}$$

**Proposition 2.** *Suppose that $[K_a, D] = -K_a$, then*

$$K_a = x^2\partial_a - 2x_a x\cdot\partial + c_1 x_a\theta\cdot\partial + c_2 x^b\theta^\alpha(\sigma_{ab})_\alpha{}^\beta\partial_\beta + c_3\theta^\alpha\theta^\beta\theta^\gamma(\sigma^{[3]})_{\alpha\beta}(\sigma_{a[3]})_\gamma{}^\delta\partial_\delta + c_4\theta^\alpha\theta^\beta\theta^\gamma(\sigma_{a[2]})_{\alpha\beta}(\sigma^{[2]})_\gamma{}^\delta\partial_\delta + c_5 x_b\theta^\alpha\theta^\beta(\sigma_a{}^{bc})_{\alpha\beta}\partial_c + c_6\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta(\sigma_{[3]})_{\alpha\beta}(\sigma^{[3]})_{\gamma\delta}\partial_a + c_7\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta(\sigma_{a[2]})_{\alpha\beta}(\sigma^{[2]b})_{\gamma\delta}\partial_b \tag{5.27}$$

*where $c_1, ..., c_7 \in \mathbb{C}$.*

**Lemma 4.5 (Siew).**

$$\theta^\alpha\theta^\beta\theta^\gamma(\sigma_{a[3]})_\gamma{}^\delta(\sigma^{[3]})_{\alpha\beta} = 0$$
$$\theta^\alpha\theta^\beta\theta^\gamma(\sigma_{a[2]})_{\alpha\beta}(\sigma^{[2]})_\gamma{}^\delta = 0$$
$$\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta(\sigma_{[3]})_{\alpha\beta}(\sigma^{[3]})_{\gamma\delta} = 0$$
$$\theta^\alpha\theta^\beta\theta^\gamma\theta^\delta(\sigma_{a[2]})_{\alpha\beta}(\sigma^{[2]b})_{\gamma\delta} = 0$$

**Corollary 4.5.1.** *We can shorten the results of Proposition 1 and Proposition 2 to*

$$S^\alpha = x^a(\sigma_a)^{\alpha\beta}Q_\beta + k_1\theta^\alpha\theta\cdot\partial + k_2(\sigma^{abc})_{\beta\gamma}(\sigma_{abc})^{\alpha\delta}\theta^\beta\theta^\gamma\partial_\delta$$
$$K_a = x^2\partial_a - 2x_a x\cdot\partial + c_1 x_a\theta\cdot\partial + c_2 x^b\theta^\alpha(\sigma_{ab})_\alpha{}^\beta\partial_\beta + c_3 x_b\theta^\alpha\theta^\beta(\sigma_a{}^{bc})_{\alpha\beta}\partial_c.$$

**Theorem 4.1.** *Let* $Q_\alpha = \partial_\alpha + k_1(\sigma^a)_{\alpha\beta}\theta^\beta\partial_a$ *and* $K_a = x^2\partial_a - 2x_a x\cdot\partial + c_1 x_a\theta\cdot\partial + c_2 x^b\theta^\alpha(\sigma_{ab})_\alpha{}^\beta\partial_\beta + c_3 x_b\theta^\alpha\theta^\beta(\sigma_a{}^{bc})_{\alpha\beta}\partial_c$ *for some constants* $k_1, c_1, c_2, c_3$ *such that* $k_1 \neq 0$. *Then* $\forall k \in \mathbb{C}\setminus\{0\} : [Q_\alpha, K_a] \neq k(\sigma_a)_{\alpha\beta}S^\beta$.

- Now relax the conditions…Only require S to be a "square-root" of K.

**Lemma 5.1.** *Suppose* $S^\alpha$ *is a fermionic vector field with units of* $\sqrt{x}$. *Then,*

$$S^\alpha = b_1\theta^\alpha x\cdot\partial + b_2 x_a(\sigma^{ab})_\beta{}^\alpha\theta^\beta\partial_b + b_3\theta^\alpha\theta\cdot\partial + b_4\theta^\beta\theta^\gamma(\sigma^{[3]})_{\beta\gamma}(\sigma_{[3]})^{\alpha\delta}\partial_\delta + b_5 x^a(\sigma_a)^{\alpha\beta}\partial_\beta$$

*for some constants* $b_1, b_2, b_3, b_4, b_5 \in \mathbb{R}$.

# NON-EXISTENCE OF S SUPERCHARGE

**Theorem 5.1.** *Suppose there is a bosonic vector field* $K_a = x^2\partial_a - 2x_a x\cdot\partial + c_1 x_a \theta\cdot\partial + c_2 x^b \theta^\alpha (\sigma_{ab})_\alpha{}^\beta \partial_\beta + c_3 x_b \theta^\alpha \theta^\beta (\sigma_a{}^{bc})_{\alpha\beta}\partial_c$ *for some* $c_1, c_2, c_3 \in \mathbb{R}$*. Then,* $\{S^\alpha, S^\beta\} \neq k(\sigma^a)^{\alpha\beta} K_a \quad \forall k \in \mathbb{R}\setminus\{0\}$.

This result is expected : According to Shnider’s ”The Superconformal Algebra in Higher Dimensions”, superconformal algebras do not exist in even dimensions $d>6$.
This is the first constructive proof that shows exactly why it does not exist in 10D.

# FUTURE RESEARCH DIRECTION

- Extend the analysis to 10+1D, where superconformal algebra is not forbidden by the no-go theorem mentioned before

- At the same time, another on-going project: Using Breitenlohner’s method to investigate supergravity.

# REFERENCES


Arvidsson, P. (2006). Superconformal theories in six dimensions, 23–24. arXiv: hep-th/0608014

de Traubenberg, M. R. (2005). Clifford algebras in physics, 6. arXiv: hep-th/0506011v1

Gates, S. J., Hu, Y., Jiang, H., & Mak, S. N. H. (2019). On linearized nordstrom supergravity in eleven and ten dimensional superspaces, 27–32. arXiv: 1812.05097v4 [hep-th]

Gates, S. J., Hu, Y., & Stiffler, K. (2019). Adinkra height yielding matrix numbers: Eigenvalue equivalence classes for minimal four-color adinkras, 4. arXiv: 1904.01738v3 [hep-th]

# QA